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Turning Waste into By-Product

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Abstract: This paper studies how the conversion of a waste stream into a useful and saleable by-product affects a firm's optimal operating strategy. We determine whether local implementation can be optimal, i.e., continuing business-as-usual to produce the original product and merely converting the collaterally generated waste stream into by-product, or whether global implementation – that re-optimizes the now joint production process – is required to maximize profit. Whereas local BPS implementation can be managed as if it were an alternate method of waste disposal, global implementation requires managerial attention at a strategic level. To determine which implementation mode is profit-maximizing, we derive optimality conditions for three possible operating regimes. These optimality conditions depend crucially on the waste disposal cost, which acts as a subsidy for the by-product that "consumes" the waste, and also on the virgin raw material cost, which acts as a subsidy for the original product that "feeds" the by-product process. These two costs create a symbiotic relationship between the original product and by-product. Since BPS turns waste into useful raw material, the firm may increase profit by generating more "waste". Although BPS is generally lauded as a win-win for business and the environment, the firm may actually increase emissions if it acts to maximize profit because it increases production to leverage the competitive advantage it gains from its operational synergy.

Keywords: Waste, by-product, environment, disposal cost, operations, competition, collateral output.

[1]Previously titled "Using By-Product Synergy for Competitive Advantage".

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1 Introduction

Accompanying the production of every product in a manufacturing process is a stream of collateral output which is usually ascribed as waste. However, pressure to improve the bottom line and increased scrutiny on environmental impact have driven firms to innovate on ways to manage their waste streams. One such alternative is the transformation of the waste stream (through further processing) into a useful and saleable by-product. The practice of converting a waste stream into by-product is commonly referred to as "by-product synergy" (BPS), a term we will use throughout this paper.[3] By practicing BPS, a manufacturer can reduce or eliminate disposal cost, generate revenue by selling the by-product, and potentially decrease its negative impact on the environment.

This paper studies how the conversion of a waste stream into by-product affects the firm's optimal operating strategy. We consider the managerial question, can "local" BPS implementation that merely converts the collaterally generated waste into by-product be optimal, or is a "global" implementation that re-optimizes the now joint production process necessary for profit-maximization? Whereas local BPS implementation can be managed as if it were an alternate method of waste disposal, global implementation requires managerial attention at a strategic level. To determine which implementation mode is profit-maximizing, we derive optimality conditions for three possible operating regimes. Additionally, we examine the environmental implications of implementing the optimal operating strategy.

The BPS concept is not new. Many examples exist in the agriculture industry, e.g., manure from livestock is used to make fertilizer, animal hide is used to make leather. Often waste water or waste heat from a plant is used to cool or heat other facilities. Although BPS is prevalent in some industries, this concept has not been universally embraced. In speaking with executives at companies that have recently implemented BPS,[4] they consistently

[3]The term "by-product synergy" is trademarked by the U.S. Business Council for Sustainable Development which defines "by-product synergy (BPS) [as] the practice of matching under-valued by-product streams with potential users, helping to create new revenues or savings for the organizations involved while simultaneously addressing social and environmental impacts." Source: www.usbcsd.org/byproductsynergy.asp.

[4]Executives include Mike Gromacki, Vice President of Engineering Loss and Control, Cook Composites and Polymers Co., and Gordon Forward, former CEO of Chaparral Steel and Chairman Emeritus of the U.S. Business Council for Sustainable Development.

mention two factors that had hindered them from implementing BPS in the past. First, application of the concept to *their own* business was a significant shift in mindset. Most managers consider their firms as producing a particular product or set of products, even though the total output from their plants includes waste streams that are generated collaterally with the products. Whereas products are managed strategically, waste is considered a manufacturing cost burden. In this "product and waste" mindset, the logical and typical approach is to minimize manufacturing cost by reducing waste. To view waste as a potential product requires a significant shift in mindset and managerial attention. Gordon Forward, former CEO of Chaparral Steel and proponent of BPS, characterized this as a shift from a "product and waste" mentality to a "100% product" mentality.[5] Thus, although BPS is prevalent in some industries (e.g., agriculture), for managers in many industries, considering their waste stream as a business opportunity instead of a liability is paradigm shift. We show that to operationalize this shift in mindset requires a shift in the firm's operating strategy.

A second hinderance to implementing BPS is the identification of potential by-products that can be produced using the waste stream. Often feasible by-products are in a completely different domain than the firm's original products, which naturally leads to the "product and waste" mentality. However, even if the firm were to embrace the "100% product" mentality, identifying feasible by-product candidates remains a challenge. This is particularly difficult for firms whose waste streams are complex and often proprietary and unique. Whereas the composition of agricultural waste streams, e.g., livestock manure, is common knowledge (or at least knowable to those who are interested), the composition of waste effluent from a chemical plant generally is not. Moreover, the chemical company may not want information about its waste stream to be publicly available because the composition (and in some cases, even the quantity) might reveal something about its proprietary manufacturing process. The by-products that can be made from complex waste streams tend to be complex, high-valued products, unlike the more obvious and prevalent examples such as manure and waste water.

Despite the challenges to implementing BPS, many companies who traditionally have not been aware of or considered this practice, in their search for win-win business and

[5]Sources: Personal conversations and http://sloanreview.mit.edu/improvisations/tag/gordon-forward/.

environment opportunities, are now looking for ways to productively use their waste streams.[6] We find that whether the opportunity ends up being a win-win situation depends on how BPS is implemented and the subsequent operating strategy of the firm. Although it is difficult to estimate the market potential of BPS opportunities, anecdotal evidence suggests that it could be significant. For example, the National Industrial Symbiosis Program, a national program established in the UK to promote BPS, helped over 400 firms save £26 million, preserving 173 jobs and creating 25 in the first 18 months of operation in the West Highlands.[7] Dow Chemical Company piloted the BPS concept within its own company. With six manufacturing plants from the Gulf Coast participating in the initial study, \$15 million of annual cost savings were identified in addition to an annual reduction of 900K MMBtu of fuel use and 108 million pounds of CO_2 emissions (US Department of Energy, Energy Efficiency and Renewal Energy 2005).

An example of a firm that has recently implemented BPS is Cook Composites and Polymers Co. (CCP). This manufacturer of high-end gel coats[8] developed a concrete coating by-product using one of its waste streams (Lee et al. 2009). CCP uses an indirect material, styrene, to clean production equipment between batches of gel coat. If the equipment is not cleaned properly, an earlier batch of gel coat could contaminate a later batch, which would lead to, at best, a defective batch of gel coat, and at worst, defects in the product onto which the gel coat is applied.[9] Styrene is a very effective cleaning agent, particularly for removing material of the composition of CCP's gel coat. Therefore, even though it is expensive to buy and also costly to dispose of after use (it is classified as a hazardous waste by the Environmental Protection Agency), CCP continues to use it. In addition to the significant

[6] To facilitate the identification of BPS opportunities, organizations around the world are organizing regional programs to bring companies together to share information about their waste streams. For example, in the UK, a national program called National Industrial Symbiosis Program (NISP) has been established to promote and facilitate BPS implementation. Within the U.S., the U.S. Business Council for Sustainable Development has organized regional BPS programs in Chicago, Houston, Kansas City, New Jersey, and Puget Sound. In each of these regional programs, between 10 and 200 companies participated by identifying and sharing information about their own waste streams and identifying potential uses for the waste streams of others. The firms signed non-disclosure agreements in order to facilitate free exchange of information.

[7] Source: http://www.nisp.org.uk.

[8] Gel coats are used as protective and aesthetic coatings for recreational vehicles, truck cabs, trailer panels, bathtubs and shower stalls, and boats.

[9] The liability cost of an undetected contaminated batch that is shipped to and used by a customer could be in the millions of dollars.

disposal cost of “rinse” styrene (styrene that has already been used to rinse/clean production equipment), CCP’s parent company, TOTAL, has a corporate goal to reduce its generation of hazardous waste.[10] Although rinse styrene flows out of CCP’s plants along with its gel coat, the styrene was considered waste and the gel coat was considered product. However, through the Kansas City Regional BPS program, CCP identified an opportunity to use its rinse styrene waste stream to produce a concrete coating by-product, an application similar in concept to its gel coat product, but sold in a completely different (lower-end, but still profitable) market.

Once a BPS process is implemented, for a plant manager whose focus is to minimize cost, a natural inclination is to continue business-as-usual to produce the original product, and merely convert the collaterally generated waste into by-product. Conversely, one might assume that since BPS clearly affects the cost structure of the manufacturing process, re-optimization of the now joint production process would be necessary. To determine whether the “local” BPS implementation inclination of the plant manager is optimal, or whether “global” implementation – that re-optimizes the entire production process – is profit-maximizing, we study a competitive setting where a firm implements BPS by converting a unique and possibly proprietary waste stream into a by-product, as in the CCP example. We assume a by-product opportunity has been identified and focus on the implementation and operational strategies of the firm. The firm can produce the by-product using the waste stream generated collaterally with its original product, or source a waste substitute virgin raw material. The firm competes in two different duopoly markets with its original product and by-product.

A symbiotic relationship between the original product and by-product is created by the disposal cost and the virgin raw material cost. When by-product is produced by “consuming” the original product’s waste stream, the waste disposal cost acts as a subsidy for the by-product because the cost is avoided. Likewise, the virgin raw material cost acts as a subsidy for the original product because its waste stream allows the firm to avoid the material cost by

[10] One of the metrics TOTAL tracks in its annual Environment and Society Report is the amount of hazardous waste generated per ton of product.

“feeding” the by-product process. We find that the implementation mode (local vs. global) and optimal operating regime (profit-maximizing equilibrium quantities) depend critically on these two “subsidies”. Moreover, these subsidies act as mechanisms to transfer wealth from one market to another through the BPS operations of the firm.

Local BPS implementation is optimal when disposal cost is low and the potential value in the by-product market is low. The firm gains at most a small cost advantage in the (relatively unattractive) by-product market, and maximizes profit by continuing business-as-usual in its original product production and converting part or all of the collaterally generated waste stream into by-product. Since the production of original product remains unchanged, the waste conversion process can be managed as if it were an alternate form of waste disposal. However, one potential danger of this mindset is that the firm might be tempted to always convert *all* the waste into by-product (an optimal operating strategy for cost minimization), when it is oftentimes profit-maximizing to still dispose of some of it. Only if the disposal cost increases to a critical threshold is it optimal to produce enough by-product to consume all the collaterally generated waste. Beyond this threshold, the firm avoids disposal cost entirely and maximizes profit by globally re-optimizing the now joint production process (global implementation). Now the cost advantage shifts to the original market where the firm can capture market share from its competitor who still bears the disposal cost. Therefore, if disposal cost is sufficiently high, waste must be managed strategically as an integrated part of the operations, not merely treated as a cost minimization effort.

Global BPS implementation is also optimal if the potential value in the by-product market is high. To capture this value, the firm can increase its by-product quantity in two ways. It can either increase production of the original product to generate more waste to feed the by-product process, or source virgin raw material to increase production above and beyond what can be produced by using the original product’s waste stream. The firm gains a cost advantage in the original market under both of these operating regimes because either the raw material subsidy lowers its cost or it avoids disposal cost that its competitor continues to bear. Therefore, it is optimal to implement BPS at a global level, resulting in increased production of the original product.

Since the waste stream is productively used in a BPS operation, an interesting implication is that the firm could actually increase profit by producing more waste per unit of original product. This is exactly opposite to the "cost minimization through waste reduction" prescription generally followed by manufacturing managers. It is in fact, a "profit maximization through waste utilization" prescription that requires managerial decision-making above the level where cost minimization is the performance metric. In the CCP example, this would translate to using more rinse styrene per batch of gel coat, possibly improving quality of the original product, and allowing more efficient production of the concrete coating by-product.

We also provide a framework for evaluating the impact on total emissions when a firm practices BPS. Although BPS is generally lauded as a win-win for business and the environment, the firm may actually increase emissions if it acts to maximize profit because it increases production to leverage the competitive advantage it gains from its operational synergy.

Literature This paper lies at the intersection of operations, economics, and the environment. As such, it builds on several streams of literature. In the operations management literature, there has been increasing interest in environmental issues (cf. Corbett and Kleindorfer 2001a,b, Kleindorfer et al. 2005, Corbett and Klassen 2006, Guide and Van Wassenhove 2006). As noted by Corbett and Klassen (2006), *"the environmental perspective extended the definition of ... defects to any form of waste."* To that end, there has been significant work done in the area of closed-loop supply chains that analyze the flow of products from the consumer back to the manufacturer or another party in the supply chain (Fleischmann et al. 1997, Guide et al. 2003, Toffel 2003, Debo et al. 2005, Guide et al. 2006, Atasu et al. 2008).

The phenomenon studied in this paper has two main distinctions from those in the existing closed-loop supply chain literature. First, BPS addresses the disposition of *pre-consumer* waste, i.e., it is the management of the waste stream from the *manufacturer*. The focus of the extant work in closed-loop supply chain is on *post-consumer* waste, i.e., the source of the waste stream is the *consumer*. The main challenges for those managing the pre-consumer waste stream are reducing disposal cost and adhering to environmental regulations or the firm's own environmental policy. In contrast, one of the main challenges for those managing the post-consumer waste stream is reverse logistics (Fleischmann et al. 1997, Jayaraman et al. 2003, Savaskan et al. 2004).

The second main distinction between BPS and remanufacturing is the nature of the quantity relationship between the original product and by-product. In a remanufacturing setting, the availability of remanufactured goods for the entire secondary market depends on the original supply from the original equipment manufacturer. Additionally, the original and remanufactured products are typically (at least) partial substitutes. Therefore, research in this area has focused on product portfolio management, e.g., balancing new and remanufactured product lines, and cannibalization (Majumder and Groenevelt 2001, Ferguson and Toktay 2006, Ferrer and Swaminathan 2006, Atasu et al. 2008). In contrast, competitors in the by-product market can produce independently of the BPS firm's waste stream. Additionally, the by-product in a BPS operation is typically completely different from and therefore sold in a completely different market than the original product. Therefore, cannibalization is not an issue: increasing the quantity of by-product does not affect the sales of the original product. Rather, in BPS, the managerial challenge is how to optimally run one process that produces two disparate, yet co-dependent products. We adhere to the perspective taken in the closed-loop supply chain literature by taking the viewpoint of the profit-maximizing firm. However, we provide a framework for analyzing how the optimal operating strategy of the firm affects the environment.

The market structure in our model is similar to multimarket oligopolies, which have been studied extensively in the economics literature. The research most closely related to ours are those papers that study firms which serve multiple markets from a single facility so that the cost structure of the products are related (cf. Bulow et al. 1985, Chen and Ross 2007). Whereas these papers investigate several cost structures (e.g., shared overhead), none capture the form used in our model which explicitly models the quantity relationship between two products that are jointly produced in a BPS operation.

"Joint production" has been studied in the environmental economics literature, however, the emphasis of this work has been on the joint production of good and bad products, i.e., production of the desired good and the associated pollutants (cf. Baumgartner and Jost 2000, Baumgartner et al. 2001). The focus of this paper is on a manufacturer that produces two desirable goods and how the synergistic quantity relationship between the two goods impacts the manufacturer's operational strategy. However, we adapt the framework presented in Baumgartner and Jost (2000) to discuss the environmental implications of the optimal operating regimes we derive. Another branch of environmental economics literature focuses on various regulatory mechanisms for reducing (typically post-consumer household) waste disposal, including disposal fees, virgin raw material tax, and recycled material content (cf. Dinan 1993, Palmer and Walls 1997, Fullerton and Wu 1998, Calcott and Walls 2000). These models take the social planner's perspective, whereas we take the perspective of the profit-maximizing firm and additionally, explicitly model the operational details of the waste conversion process.

Also related to BPS is a subfield of industrial ecology called industrial symbiosis. Much of this research describes specific examples of firms that exchange "*materials, energy, water, and/or by-products*" (Chertow 2000), and focuses on their environmental impact, e.g., the reduction in toxic emissions and landfill use, (cf. Ehrenfeld and Gertler 1997, Chertow and Lombardi 2005, Zhu et al. 2007, and van Beers et al. 2007). In this paper, we take the perspective of the profit-maximizing firm and determine, in a general context (vs. a specific example), the optimal operating strategy of a firm that implements BPS.

The rest of the paper is organized as follows. Section 2 describes our model, Section 3 presents our analysis, Section 4 examines the environmental impact of BPS, and Section 5 contains concluding remarks. All proofs are in the Appendix.

2 Model

We consider a manufacturer, firm 1, that produces product A, which we will call the original product. During the processing of the original product, waste is generated and can be further processed into a useful by-product, B. When firm 1 converts its waste stream into by-product B, we refer to this as practicing BPS. The waste from one unit of A can be used to generate $\gamma > 0$ units of B. Other raw materials may also be needed to produce B, but when firm 1 produces B using the waste stream of A, the proportion of the quantity of A to the quantity of B is 1:γ. We refer to the process that produces product A and its associated waste stream as Process $X1$ and the process for producing B as Process $Y1$. B can be produced by using A's waste stream or by purchasing virgin raw material that substitutes for A's waste stream (this allows the firm to produce B independently of A). Although technically, B is a by-product only if it is produced using A's waste stream, we will use the term by-product to refer to B even if it is produced using virgin material. Figure 1 shows the process flow diagrams for the production of products A and B in firm 1. Products A and B are completely different products and are sold into different markets.

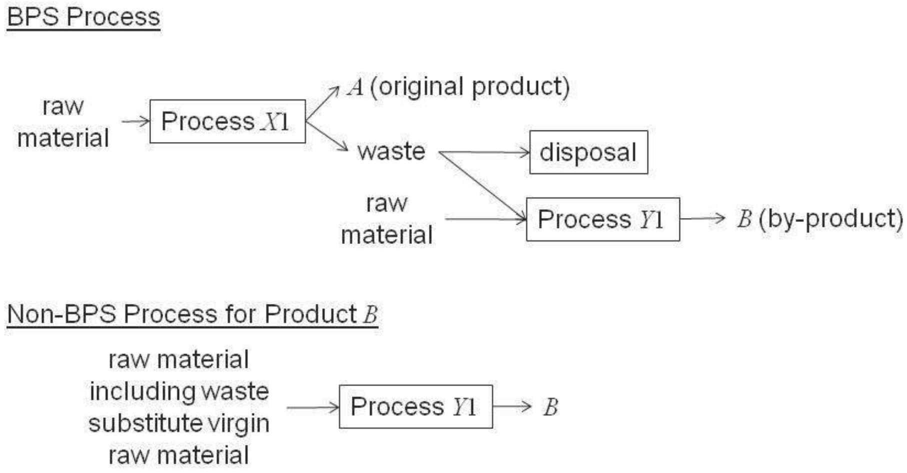

Figure 1: Process flow diagrams for firm 1's production of products A and B.

Using our CCP example from the Introduction, product A is gel coat and product B is concrete coating. The firm can either produce concrete coating (product B) using the rinse styrene waste stream from the gel coat (product A) manufacturing process, or it can purchase virgin styrene to produce concrete coating. Gel coat and concrete coating are sold into two completely different markets. Since the production process for gel coat is highly proprietary and customized, CCP and its gel coat competitors in market A each have a unique waste stream and therefore it would be unlikely that any competitor would be able to convert its waste stream into concrete coating. Therefore, we assume that firm 1 is the only firm in market A who practices BPS by converting its waste stream into product B.[11] Likewise, in the concrete coating market (market B), manufacturers use proprietary and unique processes to produce concrete coating. Given that their processes differ from each other and from CCP's by-product process, the concrete coating competitors in market B cannot use CCP's waste stream as input material, nor can they use the waste stream of CCP's competitors in the gel coat market (market A). Therefore, we assume that firm 1's waste stream can be used to produce concrete coating only in its own Process $Y1$ and that its market B competitors cannot use as input the waste stream of firm 1 or its competitors in market A.

For simplicity, we assume that firm 1 competes in duopolies in both markets.[12] That is, firm 1 competes against firms $A2$ and $B2$ in markets A and B, respectively. We assume products A and B are specialized products, therefore the firms compete on quantity, i.e., Cournot competition.[13] The demand functions in markets A and B, respectively, are $p_A = a - \alpha(q_{A1} + q_{A2})$ and $p_B = b - \beta(q_{B1} + q_{B2})$, where p_A is the price of product A in market A, q_{A1} and q_{A2} are the quantities of product A produced by firms 1 and $A2$, and $a > 0$ and $\alpha > 0$ are the characteristics of the demand curve (with analogous variables for product B: p_B, q_{B1}, q_{B2}, b, and β).

Firm 1's cost of producing one unit of A (and its collaterally generated waste stream) using Process $X1$ is c_{X1}. Firm $A2$ uses its own process incurring a cost of c_{X2} per unit of A. For both firms in market A, the disposal cost of the waste generated by the production of one unit of A is $c_w \geq 0$.[14] In market B, firm 1's competitor, firm $B2$, incurs a per unit cost of c_{B2} for producing B (which includes the cost of the unique blend of raw material used by

[11] Even if the competitors in market A had similar waste streams, firm 1 could patent the by-product conversion process. For example, Chaparral Steel patented a process for using steel slag, a steel by-product, in the production of Portland cement (see www.cemstar.com and Forward and Mangan (1999)).

[12] It can be shown that our qualitative results hold for oligopoly markets, but with considerable additional expositional complexity.

[13] Even if products A and B were commodity products (which suggest price competition), as long as the firms precommit to production capacity (i.e., the firms cannot adjust production capacity in real time), Cournot results hold (Kreps and Scheinkman 1983).

[14] Since the disposal method is usually incineration or landfill, the per unit cost of disposal should be fairly similar, even if the compositions of the waste streams differ.

firm $B2$). Firm 1 can produce B using the BPS process or independently from product A by purchasing virgin raw material (non-BPS process). If firm 1 practices BPS, it incurs a processing cost of $\frac{c_{Y1}}{\gamma}$ per unit of B, but it avoids $\frac{c_w}{\gamma}$ in disposal cost by consuming the waste stream from $\frac{1}{\gamma}$ units of A. Therefore, the cost of the BPS process is $\frac{1}{\gamma}(c_{Y1} - c_w)$ per unit of B. If firm 1 produces B using the non-BPS process, it incurs the $\frac{c_{Y1}}{\gamma}$ per unit processing cost and also a cost of $\frac{c_r}{\gamma}$ to purchase raw material to substitute for A's waste stream. Therefore, the cost of the non-BPS process is $\frac{1}{\gamma}(c_{Y1} + c_r)$ per unit of B. Since the cost of producing B using the non-BPS process is strictly higher than the cost of the BPS process, firm 1 will produce B using the non-BPS process only after it exhausts all the waste stream from A. We assume there is no quality difference between product B produced using the BPS and non-BPS processes.

Given the costs above, firm 1's profit function is

$$\Pi_1 = p_A q_{A1} + p_B q_{B1} - (c_{X1} + c_w) q_{A1} - \frac{1}{\gamma}(c_{Y1} - c_w) \min\{q_{B1}, \gamma q_{A1}\} - \frac{1}{\gamma}(c_{Y1} + c_r)(q_{B1} - \gamma q_{A1})^+ \quad (1)$$

The first two terms of Π_1 represent firm 1's revenue. The third term is the cost of producing A, which includes the disposal cost of the collaterally generated waste stream. The fourth term is the cost of producing B using the BPS process, hence the avoided cost of waste disposal is added back to the profit. The fifth term is the cost of producing B using virgin raw material, hence the cost of the waste substitute virgin raw material is subtracted from the profit.

Firm 1's competitors in markets A and B have the following profit functions:

$$\Pi_{A2} = p_A q_{A2} - c_{A2} q_{A2}, \quad (2)$$

$$\Pi_{B2} = p_B q_{B2} - c_{B2} q_{B2}, \quad (3)$$

where $c_{A2} = c_{X2} + c_w$.

3 Analysis

We first solve for the competitive equilibrium, which will define firm 1's feasible operating regimes and conditions for optimality. We then interpret these results to inform our managerial question of local vs. global BPS implementation, and discuss welfare and other operational implications.

To determine the competitive equilibrium, firm 1 chooses q_{A1} and q_{B1} to maximize the profit function expressed by (1) and its competitors choose q_{A2} and q_{B2} to maximize (2) and (3), respectively. The following lemma expresses firm 1's profit function when it produces B using the BPS process (i.e., $q_{B1} \leq \gamma q_{A1}$) and when it sources virgin raw material (i.e., $q_{B1} > \gamma q_{A1}$).

Lemma 1 *Firm 1's profit can be expressed as* $\Pi_1 = (p_A - c_{A1})q_{A1} + (p_B - c_{B1})q_{B1}$*, where* $c_{A1} = c_{X1} + c_w \equiv c^o_{A1}$ *and* $c_{B1} = \frac{1}{\gamma}(c_{Y1} - c_w) \equiv c^o_{B1}$ *if* $q_{B1} \leq \gamma q_{A1}$*, and* $c_{A1} = c_{X1} - c_r \equiv c'_{A1}$ *and* $c_{B1} = \frac{1}{\gamma}(c_{Y1} + c_r) \equiv c'_{B1}$ *if* $q_{B1} > \gamma q_{A1}$.

The disposal cost and the raw material cost create a symbiotic relationship between the by-product and the original product: c_w acts as a subsidy for the by-product, and c_r acts as a subsidy for the original product. When $q_{B1} < \gamma q_{A1}$, the firm primarily produces A and B "consumes" part of A's total waste stream. Therefore, $\frac{c_w}{\gamma}$ is subtracted from B's cost because the production of one unit of B avoids $\frac{c_w}{\gamma}$ in disposal cost for one inframarginal unit of A. When $q_{B1} > \gamma q_{A1}$, the firm primarily produces B and A partially "feeds" the by-product process with its waste stream. Therefore, c_r is subtracted from A's cost because the production of one unit of A avoids c_r in raw material cost for one inframarginal unit of B.

This straightforward lemma has an important managerial implication as cost allocation is often an issue in manufacturing. Cost allocation can affect many aspects of an organization, e.g., salesforce incentives, measures of product profitability. There are several approaches to allocating joint production costs from a cost accounting perspective (cf. Horngren et al. 2006). Toktay and Wei (2007) examine cost allocation in a manufacturing-remanufacturing operation in a non-competitive setting. We examine a new (competitive) setting, where co-dependent products are produced in one process and Lemma 1 shows how costs should be allocated to maximize profit. Allocating costs differently than what is presented in Lemma 1 would make the marginal costs for both products incorrect, thereby making the profit function incorrect, leading to suboptimal decisions.

To determine firm 1's optimal operating strategy, we compare its profit under three possible operating regimes: 1) $q_{B1} < \gamma q_{A1}$, implying $c_{A1} = c^o_{A1}$ and $c_{B1} = c^o_{B1}$, 2) $q_{B1} > \gamma q_{A1}$, implying $c_{A1} = c'_{A1}$ and $c_{B1} = c'_{B1}$, and 3) $q_{B1} = \gamma q_{A1}$, implying $c_{A1} = c^o_{A1}$, and $c_{B1} = c^o_{B1}$. In the first operating regime, only part of A's waste stream is converted into by-product B (*partial conversion*). In the second operating regime, all of A's waste stream is converted into by-product B, and additional units of B are produced using waste substitute virgin raw material (*full+ conversion*). In the third operating regime, A and B are produced in direct proportion as all of A's waste stream is converted into by-product B and B is only produced using A's waste stream (*exactly-full conversion*). The equilibrium quantities corresponding to the marginal costs implied by these three operating regimes are given in Lemmas 2, 3, and 4, respectively.

Lemma 2 *If firm 1's costs are* $c_{A1} = c^o_{A1}$ *and* $c_{B1} = c^o_{B1}$*, the equilibrium quantities are* $q^{(1)}_{A1} = \frac{a-2c^o_{A1}+c_{A2}}{3\alpha}$ *and* $q^{(1)}_{B1} = \frac{b-2c^o_{B1}+c_{B2}}{3\beta}$ *for firm 1, and* $q^{(1)}_{A2} = \frac{a-2c_{A2}+c^o_{A1}}{3\alpha}$ *and* $q^{(1)}_{B2} = \frac{b-2c_{B2}+c^o_{B1}}{3\beta}$ *for its competitors in markets* A *and* B*, respectively.*

Lemma 3 *If firm 1's costs are* $c_{A1} = c'_{A1}$ *and* $c_{B1} = c'_{B1}$*, the equilibrium quantities are* $q^{(2)}_{A1} = \frac{a-2c'_{A1}+c_{A2}}{3\alpha}$ *and* $q^{(2)}_{B1} = \frac{b-2c'_{B1}+c_{B2}}{3\beta}$ *for firm 1, and* $q^{(2)}_{A2} = \frac{a-2c_{A2}+c'_{A1}}{3\alpha}$ *and* $q^{(2)}_{B2} = \frac{b-2c_{B2}+c'_{B1}}{3\beta}$ *for its competitors in markets* A *and* B*, respectively.*

Lemma 4 *If firm 1's costs are* $c_{A1} = c^o_{A1}$ *and* $c_{B1} = c^o_{B1}$*, and* $q_{B1} = \gamma q_{A1}$*, the equilibrium quantities are* $q^{(3)}_{A1} = \rho q^{(1)}_{A1} + (1-\rho)\frac{q^{(1)}_{B1}}{\gamma}$ *and* $q^{(3)}_{B1} = \gamma q^{(3)}_{A1}$ *for firm 1, and* $q^{(3)}_{A2} = \rho q^{(1)}_{A2} + (1-\rho)\left(\frac{a-c_{A2}}{2\alpha} - \frac{q^{(1)}_{B1}}{2\gamma}\right)$ *and* $q^{(3)}_{B2} = (1-\rho)q^{(1)}_{B2} + \rho\left(\frac{b-c_{B2}}{2\beta} - \frac{\gamma q^{(1)}_{A1}}{2}\right)$ *for its competitors in markets* A *and* B*, respectively, where* $\rho \equiv \frac{\alpha}{\alpha+\beta\gamma^2} \in (0,1)$.

To determine firm 1's optimal operating strategy, we compare its profits corresponding to the scenarios given in Lemmas 2, 3, and 4. We define $\hat{q}_{A1}$, $\hat{q}_{A2}$, $\hat{q}_{B1}$, and $\hat{q}_{B2}$ to be equivalent to $q_{A1}^{(1)}$, $q_{A2}^{(1)}$, $q_{B1}^{(1)}$, and $q_{B2}^{(1)}$, respectively, when $c_w = c_r = 0$. We can interpret $\hat{q}_{A1}$, $\hat{q}_{A2}$, $\hat{q}_{B1}$, and $\hat{q}_{B2}$ to be the optimal quantities in the absence of any "subsidies". Let $\Delta \equiv \frac{\hat{q}_{B1}}{\gamma} - \hat{q}_{A1}$, $\bar{c}_w \equiv 3\alpha\Delta$, and $\bar{c}_r \equiv \frac{3\alpha\beta\gamma^2}{2(\alpha+\beta\gamma^2)}\Delta$. The following proposition gives optimality conditions for each of the three possible operating regimes.

Proposition 1 *Firm 1's profit increases after implementing BPS if and only if* $b - 2c_{B1}^o + c_{B2} > 0$. *If this condition holds, the optimal operating strategies for firm 1 and its competitors are summarized in the table below:*

Δ	c_w, c_r	q_{A1}^*, q_{B1}^*	*firm 1's optimal operating regime*	q_{A2}^*	q_{B2}^*
$\Delta \leq 0$	$0 \leq c_w \leq -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w, c_r \geq 0$	$q_{A1}^{(1)}, q_{B1}^{(1)}$	*partial conversion*	$q_{A2}^{(1)}$	$q_{B2}^{(1)}$
	$c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w \geq 0, c_r \geq 0$	$q_{A1}^{(3)}, q_{B1}^{(3)}$	*exactly-full conversion*	$q_{A2}^{(3)}$	$q_{B2}^{(3)}$
$\Delta > 0$	$\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < 1$	$q_{A1}^{(2)}, q_{B1}^{(2)}$	*full+ conversion*	$q_{A2}^{(2)}$	$q_{B2}^{(2)}$
	$\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} \geq 1$	$q_{A1}^{(3)}, q_{B1}^{(3)}$	*exactly-full conversion*	$q_{A2}^{(3)}$	$q_{B2}^{(3)}$

Whether firm 1 should implement BPS depends on the attractiveness of market B (characterized by b), and its conversion cost relative to its market B competitor (characterized by $c_{B2} - 2c_{B1}^o$). Notice that if the avoided disposal cost is high enough, the per unit cost of B could be negative, i.e., $c_{B1}^o = c_{Y1} - c_w < 0$. This condition implies that by-product production creates two revenue streams: from by-product sales and waste consumption. Therefore, implementing BPS would increase the firm's profit if the by-product market is desirable and/or the waste disposal cost is high. If BPS is profitable for the firm, the feasibility of an operating regime is determined by the potential value in the by-product market (characterized by Δ), and its optimality depends on the size of the subsidies, c_w and c_r. Figure 2 illustrates firm 1's optimal operating regime and optimal BPS implementation mode as a function of market characteristics and subsidies c_w and c_r.

Consider first the case where $\Delta \leq 0$. This condition implies that, absent the subsidies, the potential value in the by-product market is low, so that firm 1 collaterally generates more than enough waste from the production of A to make the optimal amount of B (Figure 2a). Under these market conditions, sourcing virgin raw material to produce B (full+ conversion)

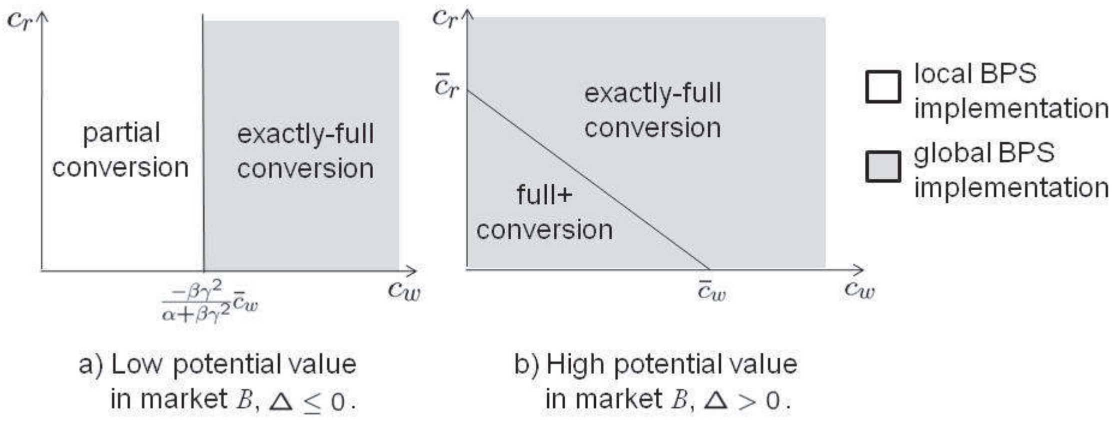

Figure 2: Optimality conditions for operating regime and BPS implementation mode.

is not feasible because market B is simply not attractive enough. Therefore, only the disposal cost determines which of the remaining two operating regimes is optimal. When disposal cost is low (i.e., $c_w \leq -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$), the subsidy for producing B (i.e., the avoided c_w cost of waste disposal) is small, making it less attractive for firm 1 to produce B. Under these conditions, local BPS implementation is profit-maximizing. Although the waste conversion process affects the cost structure of the production process, the units of B produced affect only the inframarginal units of A. Therefore, production of the original product is unaffected. This implies that the waste conversion process can be managed as if it were an alternate form of waste disposal, e.g., the plant operates as if there is no BPS process, however, instead of shipping waste to an incinerator, it is diverted elsewhere to be converted into by-product. One potential danger of this mindset is that it might be tempting to convert *all* the waste into by-product (an optimal operating strategy for cost minimization), when it is actually profit-maximizing to still dispose of some of it (partial conversion). However, if the disposal cost increases, producing A becomes more expensive and producing B becomes less expensive. Therefore, $q_{A1}^{(1)}$ and $q_{B1}^{(1)}$ respectively decrease and increase in c_w, resulting in less waste disposal. At a threshold point (i.e., $c_w = -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$), firm 1 optimally consumes all its waste in by-product production.

Beyond this point (i.e., $c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$), global BPS implementation is optimal. To maximize profit, firm 1 must increase production of its original product. For high c_w, firm 1 can capture value in both markets because of two different cost advantages. In market A, firm 1's competitor bears the high disposal cost that firm 1 now completely avoids (note, now the marginal unit of B produced affects the marginal unit of A). In market B, high c_w means firm 1's by-product is highly subsidized, but not its competitor's product. To leverage these cost advantages, firm 1 must increase production of both products in exact proportion (exactly-full conversion). This re-optimization of the now joint production process requires managerial decision-making at the strategic level since optimizing for each market separately will give a suboptimal result globally.

Consider now the case where $\Delta > 0$. This condition implies that, absent the subsidies, the potential value in the by-product market is high, so that firm 1 is unable to collaterally generate enough waste from production of A to make the optimal amount of B (Figure 2b). Obviously, under these market conditions, partial conversion is never optimal. To capture value in market B, the firm must decide whether to increase production of B by increasing production of A to generate more waste to feed the by-product process (exactly-full conversion), or source virgin raw material to produce units of B above and beyond what can be produced using A's waste stream (full+ conversion). Our results show that both operating regimes require re-optimizing the now joint production process, leading to increased production of A. The reason for the increase in A is self-evident for the exactly-full operating regime. Under full+ conversion, it is optimal to increase production of A even though virgin raw material is used to produce marginal units of B because A is being subsidized by the (avoided) raw material cost, c_r. This cost advantage allows the firm to gain market share by taking it from its competitor and also by growing the market.

Although global BPS implementation is always optimal when $\Delta > 0$, the optimal operating regime depends on the combined values of c_w and c_r. If *both* c_w and c_r are sufficiently small (i.e., $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < 1$), then full+ conversion is optimal: small c_r makes producing B using virgin raw material more attractive and small c_w makes producing B using A's waste stream less attractive. As either c_w or c_r increases, $q_{A1}^{(2)}$ increases and $q_{B1}^{(2)}$ (weakly) decreases. The effect of c_r on these two quantities is obvious since c_r increases the cost of B and acts as a subsidy for A. The optimal quantity of A increases in c_w because of firm 1's cost advantage over its competitor who continues to bear the disposal cost that the firm now completely avoids. The optimal quantity of B is constant in c_w because marginal units of B are produced using virgin material. Therefore, as c_w or c_r increases, more waste is generated and at the same time, the demand for waste decreases. Above a (joint) threshold level (i.e., $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} \geq 1$), there is enough waste from A to produce the optimal amount of B, and the optimal operating regime becomes exactly-full conversion.

An interesting operational implication of BPS is that the proportion of "waste" generated by the production of A is a measure of how efficiently the by-product is produced. Increasing the proportion parameter, γ, means that the quantity of B that can be produced per unit of A increases. The following proposition shows conditions under which firm 1 can capitalize on the increased efficiency of producing B as γ increases.

Proposition 2 *When full+ conversion is optimal, firm 1's profit increases in* γ. *When exactly-full (partial) conversion is optimal, firm 1's profit increases in* γ *if (and only if)* $c_{Y1} > c_w$.

The mechanism that translates an increase in γ into increased operational efficiency is the attenuation of the cost of producing B, $\frac{c_{B1}}{\gamma}$. When full+ conversion is optimal, $c_{B1} > 0$, therefore, increasing γ always decreases the cost. However, when exactly-full or partial conversion is optimal, each unit of by-product is subsidized by the avoided disposal cost (i.e., $c_{B1} = \frac{1}{\gamma}(c_{Y1} - c_w)$), and hence, the cost of producing B could be negative or positive. If disposal cost is lower than the processing cost (i.e., $c_w < c_{Y1}$), then $c_{B1} > 0$ and thus would be a cost in the conventional sense, i.e., the firm spends c_{B1} for every unit of B it produces. In this case, increasing γ decreases the cost of producing B, thereby making production more efficient and increasing profit. However, if the disposal cost is higher than the processing cost (i.e., $c_w \geq c_{Y1}$), the firm is actually "getting paid" to consume waste by producing B, i.e., the firm receives abs(c_{B1}) for every unit of B it produces. In this case, γ does not represent a measure of how efficiently B is produced, but rather $\frac{1}{\gamma}$ is a measure of how efficiently the waste stream of A is consumed (in the production of B). Therefore, increasing γ decreases the efficiency of waste consumption and thereby reduces the "payment" credited to each unit of B. Proposition 2 illustrates another symmetric dimension of the BPS operation. When full+ conversion is optimal, γ is a measure of the efficiency of producing B. When partial or exactly-full conversion is optimal and $c_w \geq c_{Y1}$, $\frac{1}{\gamma}$ is a measure of the efficiency of consuming/utilizing waste.

An interesting implication of Proposition 2 is that it may be optimal for the firm to *increase* the proportion of waste generated by the production of its original product. That is, under these conditions, *the firm can increase profit by generating more waste*. This is opposite to the "cost minimization through waste reduction" prescription that is generally followed by manufacturing managers. Because BPS turns waste from a liability into an asset, a "profit maximization through waste utilization" prescription is more appropriate. However, adoption of this approach requires managerial decision-making at a strategic level above the level where cost minimization is the performance metric.

While it may not always be possible for a firm to control the proportion of waste it generates, when feasible, this result uncovers a potential internal operational synergy that the firm can exploit by implementing BPS. For example, consider our gel coat manufacturer, CCP. This firm uses an expensive, hazardous chemical to clean mixing vessels between production batches. To minimize material and disposal costs, the firm strives to use as little cleaning agent as possible. However, using too little would increase the probability of contamination between batches. By converting the used cleaning agent into a saleable concrete coating by-product, CCP is able to use more cleaning agent per batch of gel coat (thereby increasing γ) to decrease the probability of contamination and also increase the combined profitability of its gel coat and concrete coating products (Lee et al. 2009).

Welfare Implications As shown in Proposition 1, the disposal cost and virgin raw material cost are critical in determining which operating regime is optimal. We now examine how value is created and destroyed in the original market and by-product market as these two costs change. The following two lemmas show how the optimal quantities of the firms change as a function of these two costs. We use these results in Propositions 3 and 4 to show how c_w and c_r affect market size.

Lemma 5 *For $\Delta \leq 0$, the optimal quantities are independent of c_r and change with respect to c_w in the following ways:*

- *Firm 1:* $\frac{\partial q_{A1}^{(1)}}{\partial c_w} < 0$, $\frac{\partial q_{A1}^{(3)}}{\partial c_w} > 0$; $\frac{\partial q_{B1}^{(1)}}{\partial c_w} > \frac{\partial q_{B1}^{(3)}}{\partial c_w} > 0$
- *Firm A2:* $\frac{\partial q_{A2}^{(3)}}{\partial c_w} < \frac{\partial q_{A2}^{(1)}}{\partial c_w} < 0$
- *Firm B2:* $\frac{\partial q_{B2}^{(1)}}{\partial c_w} < \frac{\partial q_{B2}^{(3)}}{\partial c_w} < 0$

If $c_w = -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$, then $q_{A1}^{(1)} = q_{A1}^{(3)}$, $q_{B1}^{(1)} = q_{B1}^{(3)}$, $q_{A2}^{(1)} = q_{A2}^{(3)}$, and $q_{B2}^{(1)} = q_{B2}^{(3)}$.

Lemma 6 *For $\Delta > 0$, the optimal quantities change with respect to c_w and c_r in the following ways:*

- *Firm 1:* $\frac{\partial q_{A1}^{(2)}}{\partial c_w} > \frac{\partial q_{A1}^{(3)}}{\partial c_w} > 0$, $\frac{\partial q_{A1}^{(2)}}{\partial c_r} > \frac{\partial q_{A1}^{(3)}}{\partial c_r} = 0$; $\frac{\partial q_{B1}^{(3)}}{\partial c_w} > \frac{\partial q_{B1}^{(2)}}{\partial c_w} = 0$, $\frac{\partial q_{B1}^{(2)}}{\partial c_r} < \frac{\partial q_{B1}^{(3)}}{\partial c_r} = 0$
- *Firm A2:* $\frac{\partial q_{A2}^{(2)}}{\partial c_w} < \frac{\partial q_{A2}^{(3)}}{\partial c_w} < 0$, $\frac{\partial q_{A2}^{(2)}}{\partial c_r} < \frac{\partial q_{A2}^{(3)}}{\partial c_r} = 0$
- *Firm B2:* $\frac{\partial q_{B2}^{(3)}}{\partial c_w} < \frac{\partial q_{B2}^{(2)}}{\partial c_w} = 0$, $\frac{\partial q_{B2}^{(2)}}{\partial c_r} > \frac{\partial q_{B2}^{(3)}}{\partial c_r} = 0$

If $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} = 1$, then $q_{A1}^{(2)} = q_{A1}^{(3)}$, $q_{B1}^{(2)} = q_{B1}^{(3)}$, $q_{A2}^{(2)} = q_{A2}^{(3)}$, and $q_{B2}^{(2)} = q_{B2}^{(3)}$.

The following two propositions show how the market sizes change as a function of disposal cost and virgin raw material cost.

Proposition 3 *The size of market A decreases in c_w. The size of market B increases in c_w if $\Delta \leq 0$ and weakly increases in c_w if $\Delta > 0$.*

Proposition 4 *If $\Delta > 0$, the size of market A weakly increases in c_r and the size of market B weakly decreases in c_r. If $\Delta \leq 0$, c_r does not affect the market sizes.*

Increasing disposal cost has inverse effects on the sizes of markets A and B and increasing c_r has the opposite inverse effects on the market sizes. Propositions 3 and 4 again illustrate the symmetry of c_w and c_r acting as subsidies for the by-product and original product, respectively. These two subsidies essentially act as mechanisms to transfer wealth from one market to another.

Consider the c_w subsidy. When local BPS implementation (partial conversion) is optimal, increasing disposal cost decreases the firm's production of A and increases its production of B, leading to value destruction in market A and value creation in market B. Essentially, the disposal cost enables a transfer of wealth from market A to market B. Notice that increasing disposal cost in this operating regime acts to reduce waste disposal in two ways: 1) it reduces the total waste that is produced in market A, and 2) of the waste that is still produced, more of it is converted to by-product. Therefore, using disposal cost as a regulatory mechanism for reducing waste disposal has impact on both the (reduced) generation and (increased) utilization of waste. Moreover, increasing disposal cost rewards the firm practicing BPS.

When global BPS implementation is optimal, even though all of firm 1's waste is consumed in by-product production, increasing c_w still destroys value in market A by increasing the cost of firm 1's competitor.[15] Under exactly-full conversion, value is created in market B by decreasing firm 1's cost (thereby increasing total quantity). However, under full+ conversion, since the marginal unit of B is produced using virgin raw material, the optimal quantity is independent of c_w. Therefore, in this operating regime, increasing c_w merely destroys value in market A without creating value in market B.

The c_r subsidy works similarly to the c_w subsidy, except that c_r subsidizes the original product and increases the cost of the by-product. Therefore, under full+ conversion, increasing c_r increases the production of A and decreases the production of B. This facilitates a transfer of wealth from market B to market A. The reason why c_r has no impact on the market sizes when $\Delta \leq 0$ is because, unlike c_w, firm 1's competitor is not affected by c_r. Therefore, since firm 1 does not source virgin raw material when $\Delta \leq 0$, c_r does not affect the optimal strategies (Lemma 5).

4 Environmental Impact

Although BPS is generally viewed as a positive impact on the environment, careful analysis shows that its effect on the environment is ambiguous. We focus here on the impact on

[15] Note again that increasing c_w means less waste is produced by firm $A2$ and in total.

emissions (versus, for example, the environmental impact of waste disposal in landfill) when a firm practices BPS. Two environmental benefits of BPS that are often highlighted are: 1) waste disposal is avoided, e.g., less incineration, and 2) the by-product replaces competing products in market B that are produced in a manner that is worse for the environment, e.g., the displaced products use virgin raw material which creates emissions during the extraction process. We show in the following analysis that the environmental impact of BPS depends on the optimal operating regime. Specifically, in order to understand the impact of BPS on emissions, we need to consider the change in the overall quantity of A and B produced in the two markets and also consider how they are produced.

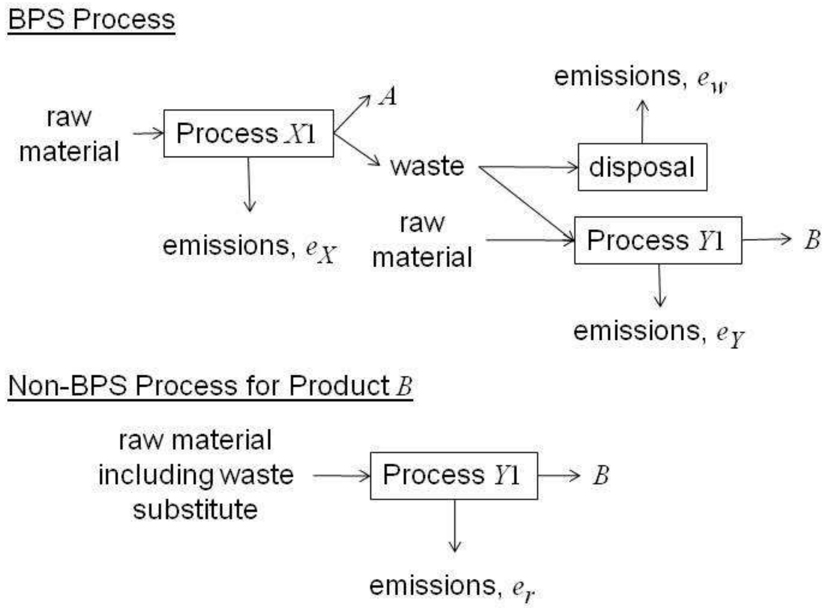

Figure 3: Emissions from firm 1's operations.

We have adapted the framework presented in Baumgartner and Jost (2000) to model emissions from firm 1's manufacturing process. Figure 3 shows the sources of potential emissions from firm 1. Firm $A2$ in market A emits through its own Process $X2$ and waste disposal, and firm $B2$ in market B emits through the process it uses to produce B. The total emissions from the production of products A and B in the two markets is:

$$E = e_X(q_{A1}+q_{A2}) + e_w((q_{A1}-\frac{q_{B1}}{\gamma})^+ + q_{A2}) + e_Y \min\{\gamma q_{A1}, q_{B1}\} + e_r(q_{B1}-\gamma q_{A1})^+ + e_B q_{B2},$$

where

- e_X is the emissions from Processes $X1$ and $X2$ for producing one unit of A (we assume that firms producing A have the same per unit emissions level),
- e_w is the emissions from the disposal of waste generated by production of one unit of A (we assume that firms producing A use the same method of waste disposal, e.g., incineration),
- e_Y is the emissions from the BPS version of Process $Y1$ for converting the waste from $\frac{1}{\gamma}$ units of A into one unit of B (this includes emissions from raw materials extraction *except* for the raw material that A's waste stream replaces),
- e_r is the emissions from the non-BPS version of Process $Y1$ (this includes emissions from *all* raw materials extraction), and
- e_B is the emissions from the process used by firm $B2$ for producing one unit of B (this includes the emissions from all raw materials extraction).

Note that if $e_Y < e_B$, this would imply that firm 1's BPS process for producing B emits less than its competitor's process. This is a common assumption for BPS processes since the use of waste as an input typically reduces the use of virgin raw material thereby avoiding the emissions from the extraction process. We additionally define $q_{B2}^{(0)} \equiv \frac{b-c_{B2}}{2\beta}$ which is firm $B2$'s optimal quantity before firm 1 enters market B, i.e., $q_{B2}^{(0)}$ is the monopolist quantity. Let $E^{(0)}$ be the total emissions level before firm 1 implements BPS. Let $E^{(1)}$, $E^{(2)}$, and $E^{(3)}$ be the emissions levels when partial, full+, and exactly-full conversion, respectively, are optimal. The expressions for total emissions are given by:

$$
\begin{aligned}
E^{(0)} &= e_X(q_{A1}^{(1)} + q_{A2}^{(1)}) + e_w(q_{A1}^{(1)} + q_{A2}^{(1)}) + e_B q_{B2}^{(0)} \\
E^{(1)} &= e_X(q_{A1}^{(1)} + q_{A2}^{(1)}) + e_w(q_{A1}^{(1)} - \frac{q_{B1}^{(1)}}{\gamma} + q_{A2}^{(1)}) + e_Y q_{B1}^{(1)} + e_B q_{B2}^{(1)} \\
E^{(2)} &= e_X(q_{A1}^{(2)} + q_{A2}^{(2)}) + e_w q_{A2}^{(2)} + e_Y \gamma q_{A1}^{(2)} + e_r(q_{B1}^{(2)} - \gamma q_{A1}^{(2)}) + e_B q_{B2}^{(2)} \\
E^{(3)} &= e_X(q_{A1}^{(3)} + q_{A2}^{(3)}) + e_w q_{A2}^{(3)} + e_Y q_{B1}^{(3)} + e_B q_{B2}^{(3)}
\end{aligned}
$$

The following proposition shows the directional change in each type of emissions after firm 1 implements BPS.

Proposition 5 *The changes in emissions e_X, e_w, e_Y, e_r, and e_B after firm 1 implements BPS are summarized in the table below:*

operating regime	e_X	e_w	e_Y	e_r	e_B
partial conversion	0	−	+	0	−
full+ conversion	+	−	+	+	−
exactly-full conversion	+	−	+	0	−

Consider first the result for partial conversion. Clearly, e_w emissions decreases because firm 1 avoids waste disposal for some of its A production, and nothing else changes in market A. In market B, firm 1 takes market share from its competitor, but it also grows the market, i.e., $q_{B1}^{(1)} + q_{B2}^{(1)} - q_{B2}^{(0)} > 0$. Therefore, even if the BPS process for producing B is less polluting than firm $B2$'s process, i.e., $e_Y < e_B$, that would not guarantee that emissions in market B would decrease when firm 1 implements BPS. The per unit emissions from the BPS process would have to be small enough to also compensate for the additional emissions generated by the growth of market B, i.e., $e_Y < \frac{q_{B2}^{(0)} - q_{B2}^{(1)}}{q_{B1}^{(1)}} e_B < e_B$.

Under full+ and exactly-full conversion, market B again grows as firm 1 implements BPS. Again, the e_Y, and for full+ conversion, e_r, emissions not only have to be lower than e_B, but they have to be sufficiently low to compensate for the increase in total B production. In market A, e_w emissions decreases, as expected, but through two mechanisms: 1) firm 1 avoids waste disposal (and hence emissions) using the BPS process, and 2) in a competitive response to firm 1's cost advantage, its competitor decreases production (and hence emissions). However, firm 1 capitalizes on its cost advantage by increasing production of A, thereby generating more e_X emissions. In order for emissions in market A to decrease, the decrease in e_w emissions must outweigh the increase in e_X emissions, i.e., $e_w > \frac{q_{A1}^{(2)} + q_{A2}^{(2)} - q_{A1}^{(1)} - q_{A2}^{(1)}}{q_{A1}^{(1)} + q_{A2}^{(1)} - q_{A2}^{(2)}} e_X$.

Although competition effects complicate the BPS emissions analysis, a general effect of implementing BPS is that emissions from firm 1's competitors decrease. The symbiotic relationship created by the two cost subsidies gives firm 1 an advantage in both markets allowing it to take market share from its competitors. Therefore, the competitors' production, and hence emissions, decrease. As a result, if firm 1 reduces its own emissions after implementing BPS, total emissions in the two markets will decrease. This result is formalized in the following corollary.

Corollary 1 *Emissions from firm 1's competitors (weakly) decrease after firm 1 implements BPS. Therefore, if firm 1 decreases its own emissions, total emissions from markets A and B decreases.*

Notice that there is no (positive or negative) correlation between what is profit-maximizing for the firm and what is best for the environment. Implementing BPS could be a win-win for business and the environment, but it could also force a tradeoff. What is interesting is that a sub-optimal BPS implementation could be best for the environment. For example, local implementation that is suboptimal may reduce emissions, whereas profit-maximizing global implementation (that increases production of both A and B) may increase emissions. As with many environmental issues, the impact of a seemingly benign or even beneficial concept such as utilizing a waste stream in reality becomes quite ambiguous and complex, and may end up forcing a tradeoff between business and environmental objectives.

5 Concluding Remarks

The practice of BPS is the operationalization of the proverb, *"One man's garbage is another man's treasure".* Waste has traditionally been treated as a source of operational inefficiency or cost. As such, cost minimization through waste reduction has been a standard approach taken to improve operational performance. BPS offers a new lens that allows a firm to turn a liability into a potential opportunity. Consistent with a "product and waste" mindset, a firm can implement BPS at a local level and simply convert its collaterally generated waste stream into by-product. This implementation mode can be managed as if it were an alternate form of waste disposal. Alternatively, the firm can implement BPS at a global level, which requires re-optimizing its entire production process. This latter approach necessarily requires a shift to a "100% product" mindset that requires managerial attention at a strategic level. In this paper, we showed under what conditions local vs. global BPS implementation is profit-maximizing.

We studied a setting where a firm converts a complex, possibly proprietary waste stream into a (typically non-commodity) by-product. We found that a symbiotic relationship between the by-product and the original product is created by the waste disposal cost, which acts as a subsidy for the by-product, and the virgin raw material cost, which acts as a subsidy for the original product. The waste disposal cost is critical in determining which implementation mode is optimal because it creates competitive effects in both markets: in the original market, increasing disposal cost hurts the firm's competitor (by increasing its cost burden), and in the by-product market, increasing disposal cost benefits the firm (by reducing its cost burden). When disposal cost is low, the competitive effects in both markets are low. Therefore, local BPS implementation is optimal – the firm maximizes profit by continuing business-as-usual to produce its original product, and merely converting (sometimes only part of) its waste stream into by-product. If disposal cost is high, the marginal unit of by-product has high competitive impact in both markets, therefore global BPS implementation that re-optimizes the entire process is profit-maximizing.

If the potential value in the by-product market is high. It may be profit-maximizing for the firm to source virgin raw material to produce by-product above and beyond the quantity that can be produced by using the original product's waste stream. Even in this case, when the marginal unit of by-product does not consume waste, re-optimizing the production process is optimal. This is because the virgin raw material cost acts as a subsidy for the original product since its waste stream "feeds" the by-product process and allows the firm to avoid this cost. The subsidized original product now has a cost advantage and captures market share by taking it from its competitor and also by growing the original market.

An interesting implication of the joint and synergistic nature of a BPS operation is that the proportion of waste generated by the original product can be interpreted as a measure of how efficiently the by-product is produced. That is, the more waste that is produced per unit of original product, the more efficient the production of by-product is. Therefore, the firm could increase profit by generating more "waste".

The general perception is that BPS is beneficial for the environment. However, we showed that the net environmental impact depends on the optimal operating regime. The firm's cost advantage from its BPS operation allows it to take market share from its competitors, thereby lowering their emissions (through quantity reduction). Therefore, if the firm can reduce its own emissions by implementing BPS, total emissions decreases. However, if the firm optimizes its operation, then its own production quantities increase. Whether the firm's emissions decreases depends on how polluting the production processes are relative to the avoided waste disposal process.

Limitations and Extensions In this paper, we used a simple model to determine the optimal operating regime of a firm practicing BPS in a competitive setting. In particular, we examined a market structure where the manufacturer producing the waste also converted it into by-product. Another feasible market structure is one where the manufacturer pays a third party to do the conversion. This payment would obviously have to be less than the disposal cost, however, there may be economies of scale that allow the third party to more efficiently process the by-product. There are a set of interesting issues surrounding the question of vertical integration, e.g., cost allocation and pricing, that would be a fruitful avenue for future research.

In the realm of waste disposal always lurks the shadow of regulation. The Environmental Protection Agency regulates how hazardous wastes must be disposed of. It is unclear if a given BPS implementation involving hazardous waste would adhere to the existing regulations. If regulators want to promote productive use of waste streams, there will have to be a careful balance between encouraging BPS innovation that is good for business and the environment, and curtailing illegal waste disposal disguised as by-product sales. The interaction between market mechanisms and regulations in the BPS context is an important and interesting area for future research.

We feel that the practice of productively using waste is an important concept that warrants further research as it has both strategic benefits for the firm and potentially beneficial impact on the environment. In particular, because of the unique operational implications of using waste, the field of operations management is well-suited to uncover managerial insights in this area. We hope this paper helps to lay a foundation for future work.

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Appendix A Proofs

Proof of Lemma 1

If $q_{B1} \leq \gamma q_{A1}$, the last term in equation (1) drops out. Rearranging the remaining terms gives $\Pi_1 = p_A q_{A1} + p_B q_{B1} - (c_{X1} + c_w) q_{A1} - \frac{1}{\gamma}(c_{Y1} - c_w) q_{B1} = (p_A - c^o_{A1}) q_{A1} + (p_B - c^o_{B1}) q_{B1}$, which gives us the first result of the lemma. If $q_{B1} > \gamma q_{A1}$, equation (1) becomes

$$\begin{aligned}
\Pi_1 &= p_A q_{A1} + p_B q_{B1} - (c_{X1} + c_w) q_{A1} - \frac{1}{\gamma}(c_{Y1} - c_w)\gamma q_{A1} - \frac{1}{\gamma}(c_{Y1} + c_r)(q_{B1} - \gamma q_{A1}) \\
&= p_A q_{A1} + p_B q_{B1} - (c_{X1} - c_r) q_{A1} - \frac{1}{\gamma}(c_{Y1} + c_r) q_{B1} \\
&= (p_A - c'_{A1}) q_{A1} + (p_B - c'_{B1}) q_{B1},
\end{aligned}$$

which gives us the second result of the lemma and completes the proof. ■

Proof of Lemmas 2 and 3

Firm 1 maximizes $\Pi_1 = (p_A - c_{A1}) q_{A1} + (p_B - c_{B1}) q_{B1}$ where $c_{A1} = c^o_{A1}$ and $c_{B1} = c^o_{B1}$ in Lemma 2 and $c_{A1} = c'_{A1}$ and $c_{B1} = c'_{B1}$ in Lemma 3. Substituting $p_A = a - \alpha(q_{A1} + q_{A2})$ and $p_B = b - \beta(q_{B1} + q_{B2})$ and differentiating with respect to q_{A1} and q_{B1} gives first order conditions:

$$\begin{array}{ll}
\frac{\partial \Pi_1}{\partial q_{A1}} = a - c_{A1} - \alpha q_{A2} - 2\alpha q_{A1} = 0 & \frac{\partial \Pi_1}{\partial q_{B1}} = b - c_{B1} - \beta q_{B2} - 2\beta q_{B1} = 0 \\
q^*_{A1} = \frac{a - c_{A1} - \alpha q_{A2}}{2\alpha} & q^*_{B1} = \frac{b - c_{B1} - \beta q_{B2}}{2\beta}
\end{array}$$

The second order conditions are negative, therefore FOCs are necessary and sufficient.

Firm 1's competitor in market A maximizes $\Pi_{A2} = (p_A - c_{A2}) q_{A2}$, which leads to a first order condition $q^*_{A2} = \frac{a - c_{A2} - \alpha q_{A1}}{2\alpha}$. Substituting q^*_{A2} back into q^*_{A1} above gives optimal quantities $q^*_{A1} = \frac{a - 2c_{A1} + c_{A2}}{3\alpha}$ and $q^*_{A2} = \frac{a - 2c_{A2} + c_{A1}}{3\alpha}$, which are equal to $q^{(1)}_{A1}$ and $q^{(1)}_{A2}$ in Lemma 2 and $q^{(2)}_{A1}$ and $q^{(2)}_{A2}$ in Lemma 3, respectively. A similar analysis can be done to derive $q^*_{B1} = \frac{b - 2c_{B1} + c_{B2}}{3\beta}$ and $q^*_{B2} = \frac{b - 2c_{B1} + c_{B1}}{3\beta}$, which are equal to $q^{(1)}_{B1}$ and $q^{(1)}_{B2}$ in Lemma 2 and $q^{(2)}_{B1}$ and $q^{(2)}_{B2}$ in Lemma 3, respectively. This completes the proof. ■

Proof of Lemma 4

We substitute $c_{A1} = c^o_{A1}$, $c_{B1} = c^o_{B1}$, and $q_{B1} = \gamma q_{A1}$ into firm 1's profit function $\Pi_1 = (p_A - c_{A1}) q_{A1} + (p_B - c_{B1}) q_{B1}$ to give:

$$\begin{aligned}
\Pi_1 &= (a - \alpha(q_{A1} + q_{A2}) - c^o_{A1}) q_{A1} + (b - \beta(\gamma q_{A1} + q_{B2}) - c^o_{B1}) \gamma q_{A1} \\
&= (a - c^o_{A1} + \gamma(b - c^o_{B1}) - \alpha q_{A2} - \beta\gamma q_{B2}) q_{A1} - (\alpha + \beta\gamma^2) q^2_{A1}.
\end{aligned}$$

The first order condition is:

$$a - c^o_{A1} + \gamma(b - c^o_{B1}) - \alpha q_{A2} - \beta\gamma q_{B2} - 2(\alpha + \beta\gamma^2)q_{A1} \;=\; 0,$$

$$\text{which implies } q^{(3)}_{A1} = \frac{a - c^o_{A1} + \gamma(b - c^o_{B1}) - \alpha q_{A2} - \beta\gamma q_{B2}}{2(\alpha + \beta\gamma^2)}.$$

The second order condition is negative, therefore the FOC is necessary and sufficient.

Competitor response functions are $q_{A2} = \frac{a - c_{A2} - \alpha q_{A1}}{2\alpha}$ and $q_{B2} = \frac{b - c_{B2} - \beta\gamma q_{A1}}{2\beta}$. Substituting these back into $q^{(3)}_{A1}$ gives firm 1's optimal quantity:

$$\begin{aligned} q^{(3)}_{A1} &= \frac{a - 2c^o_{A1} + c_{A2} + \gamma(b - 2c^o_{B1} + c_{B2})}{3(\alpha + \beta\gamma^2)} \\ &= \frac{\alpha}{\alpha + \beta\gamma^2} q^{(1)}_{A1} + \frac{\beta\gamma^2}{\alpha + \beta\gamma^2}\left(\frac{q^{(1)}_{B1}}{\gamma}\right) \\ &= \rho q^{(1)}_{A1} + (1 - \rho)\frac{q^{(1)}_{B1}}{\gamma}. \end{aligned}$$

By assumption, $q^{(3)}_{B1} = \gamma q^{(3)}_{A1}$.

Substituting $q^{(3)}_{A1}$ into the competitors' response functions, $q_{A2} = \frac{a - c_{A2} - \alpha q_{A1}}{2\alpha}$ and $q_{B2} = \frac{b - c_{B2} - \beta\gamma q_{A1}}{2\beta}$, gives $q^{(3)}_{A2} = \rho q^{(1)}_{A2} + (1-\rho)\left(\frac{a - c_{A2}}{2\alpha} - \frac{q^{(1)}_{B1}}{2\gamma}\right)$ and $q^{(3)}_{B2} = (1-\rho)q^{(1)}_{B2} + \rho\left(\frac{b - c_{B2}}{2\beta} - \frac{\gamma q^{(1)}_{A1}}{2}\right)$, respectively. This completes the proof. ■

Proof of Proposition 1

To solve for firm 1's optimal quantities to maximize

$$\Pi_1 \;=\; p_A q_{A1} + p_B q_{B1} - (c_{X1} + c_w)q_{A1} - \frac{1}{\gamma}(c_{Y1} - c_w)\min\{q_{B1}, \gamma q_{A1}\} - \frac{1}{\gamma}(c_{Y1} + c_r)(q_{B1} - \gamma q_{A1})^+,$$

we consider three cases (corresponding to Lemmas 2, 3, and 4, respectively): 1) $q_{B1} < \gamma q_{A1}$ (partial conversion), 2) $q_{B1} > \gamma q_{A1}$ (full+ conversion), and 3) $q_{B1} = \gamma q_{A1}$ (exactly-full conversion). Let $\Pi^{(1)}_1$, $\Pi^{(2)}_1$, and $\Pi^{(3)}_1$ be firm 1's profit under the conditions in Lemmas 2, 3, and 4, respectively. We first determine which operating regime is profit-maximizing, conditional on the firm implementing BPS. Then, using this result, we determine whether implementing BPS increases the firm's profit.

If $q^{(1)}_{B1} \leq \gamma q^{(1)}_{A1}$, then the optimal quantities are $q^*_{A1} = q^{(1)}_{A1}$ and $q^*_{B1} = q^{(1)}_{B1}$ because either $q_{B1} \leq \gamma q_{A1}$ and $q^{(1)}_{A1}$, $q^{(1)}_{B1}$ maximize $\Pi^{(1)}_1$ (Lemma 2), or $q_{B1} > \gamma q_{A1}$ which means that $\Pi^{(2)}_1$ is the profit function. However, note that $\Pi^{(1)}_1 > \Pi^{(2)}_1$ when $q_{B1} > \gamma q_{A1}$. Therefore, $q^{(1)}_{A1}$ and $q^{(1)}_{B1}$ are the global argmax.

If $q^{(1)}_{B1} > \gamma q^{(1)}_{A1}$ and $q^{(2)}_{B1} < \gamma q^{(2)}_{A1}$, then $q^*_{A1} = q^{(3)}_{A1}$ and $q^*_{B1} = q^{(3)}_{B1}$ because the constraint on $\Pi^{(1)}_1$ (i.e., $q^{(1)}_{B1} \leq \gamma q^{(1)}_{A1}$) is binding and the constraint on $\Pi^{(2)}_1$ (i.e., $q^{(1)}_{B1} \geq \gamma q^{(1)}_{A1}$) is also binding. Therefore, $q^*_{A1} = q^{(3)}_{A1} = \gamma q^{(3)}_{B1} = q^*_{B1}$.

If $q^{(1)}_{B1} > \gamma q^{(1)}_{A1}$ and $q^{(2)}_{B1} \geq \gamma q^{(2)}_{A1}$, then we need to compare $\Pi^{(2)}_1$ and $\Pi^{(3)}_1$ because $\Pi^{(3)}_1$ is $\Pi^{(1)}_1$ optimized under the constraint $q_{B1} = \gamma q_{A1}$, and we know that $\Pi^{(1)}_1 > \Pi^{(2)}_1$ when $q_{B1} > \gamma q_{A1}$. Therefore, we need to compare $\Pi^{(3)}_1$ (which is $\Pi^{(1)}_1$ at the boundary condition $q_{B1} = \gamma q_{A1}$)

and $\Pi_1^{(2)}$.

First, note that firm 1's profit can be expressed as:

$$\begin{aligned}
\Pi_1 &= (p_A - c_{A1})q_{A1} + (p_B - c_{B1})q_{B1} \\
&= (a - \alpha(q_{A1} + q_{A2}) - c_{A1})q_{A1} + (b - \beta(q_{B1} + q_{B2}) - c_{B1})q_{B1} \\
&= (a - c_{A1} - \alpha(\frac{a - c_{A2} - \alpha q_{A1}}{2\alpha}))q_{A1} - \alpha q_{A1}^2 + (b - c_{B1} - \beta(\frac{b - c_{B2} - \beta q_{B1}}{2\beta}))q_{B1} - \beta q_{B1}^2 \\
&= \frac{3\alpha}{2}(\frac{a - 2c_{A1} + c_{A2}}{3\alpha})q_{A1} - \frac{\alpha}{2}q_{A1}^2 + \frac{3\beta}{2}(\frac{b - 2c_{B1} + c_{B2}}{3\beta})q_{B1} - \frac{\beta}{2}q_{B1}^2 \\
&= \alpha q_{A1}^2 + \beta q_{B1}^2 \qquad \text{(A1)}
\end{aligned}$$

We show below that $\Pi_1^{(2)} - \Pi_1^{(3)} > 0$ when $q_{B1}^{(1)} > \gamma q_{A1}^{(1)}$:

$$\begin{aligned}
\Pi_1^{(2)} - \Pi_1^{(3)} &= \alpha(q_{A1}^{(2)})^2 + \beta(q_{B1}^{(2)})^2 - \alpha(q_{A1}^{(3)})^2 - \beta(q_{B1}^{(3)})^2 \\
&= \alpha(q_{A1}^{(2)} - q_{A1}^{(3)})(q_{A1}^{(2)} + q_{A1}^{(3)}) + \beta(q_{B1}^{(2)} - q_{B1}^{(3)})(q_{B1}^{(2)} + q_{B1}^{(3)}) \\
&= \alpha\left(\frac{\beta\gamma^2}{\alpha+\beta\gamma^2}(q_{A1}^{(1)} - \frac{q_{B1}^{(1)}}{\gamma}) + \frac{2(c_w + c_r)}{3\alpha}\right)\left(\frac{2\alpha+\beta\gamma^2}{\alpha+\beta\gamma^2}q_{A1}^{(1)} + \frac{\beta\gamma^2}{\alpha+\beta\gamma^2}\frac{q_{B1}^{(1)}}{\gamma} + \frac{2(c_w + c_r)}{3\alpha}\right) \\
&\quad +\beta\left(\frac{\alpha\gamma}{\alpha+\beta\gamma^2}(\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)}) - \frac{2(c_w + c_r)}{3\beta\gamma}\right)\left(\frac{\alpha\gamma}{\alpha+\beta\gamma^2}q_{A1}^{(1)} + \frac{\alpha+2\beta\gamma^2}{\alpha+\beta\gamma^2}q_{B1}^{(1)} - \frac{2(c_w + c_r)}{3\beta\gamma}\right) \\
&= \frac{\alpha\beta\gamma^2}{\alpha+\beta\gamma^2}(q_{A1}^{(1)} - \frac{q_{B1}^{(1)}}{\gamma})(\frac{2\alpha+\beta\gamma^2}{\alpha+\beta\gamma^2}q_{A1}^{(1)} + \frac{\beta\gamma^2}{\alpha+\beta\gamma^2}\frac{q_{B1}^{(1)}}{\gamma}) + \frac{4(c_w + c_r)}{3}q_{A1}^{(1)} + \frac{4(c_w + c_r)^2}{9\alpha} \\
&\quad -\frac{\alpha\beta\gamma^2}{\alpha+\beta\gamma^2}(q_{A1}^{(1)} - \frac{q_{B1}^{(1)}}{\gamma})(\frac{\alpha}{\alpha+\beta\gamma^2}q_{A1}^{(1)} + \frac{\alpha+2\beta\gamma^2}{\alpha+\beta\gamma^2}\frac{q_{B1}^{(1)}}{\gamma}) - \frac{4(c_w + c_r)}{3}\frac{q_{B1}^{(1)}}{\gamma} + \frac{4(c_w + c_r)^2}{9\beta\gamma^2} \\
&= \frac{\alpha\beta\gamma^2}{\alpha+\beta\gamma^2}(q_{A1}^{(1)} - \frac{q_{B1}^{(1)}}{\gamma})^2 + \frac{4}{3}(c_w + c_r)(q_{A1}^{(1)} - \frac{q_{B1}^{(1)}}{\gamma}) + \frac{4}{9}(c_w + c_r)^2\frac{\alpha+\beta\gamma^2}{\alpha\beta\gamma^2} \\
&= \frac{\alpha\beta\gamma^2}{\alpha+\beta\gamma^2}(\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)})^2 - \frac{4}{3}(c_w + c_r)(\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)}) + \frac{4}{9}(c_w + c_r)^2\frac{\alpha+\beta\gamma^2}{\alpha\beta\gamma^2},
\end{aligned}$$

which is quadratic in $\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)}$. We know that $\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)} > 0$ by assumption. We apply the quadratic formula to find the roots of $\Pi_1^{(2)} - \Pi_1^{(3)}$:

$$\frac{\frac{4(c_w+c_r)}{3} \pm \sqrt{\frac{16(c_w+c_r)^2}{9} - \frac{4\alpha\beta\gamma^2}{\alpha+\beta\gamma^2}\frac{4(c_w+c_r)^2}{9}\frac{\alpha+\beta\gamma^2}{\alpha\beta\gamma^2}}}{\frac{2\alpha\beta\gamma^2}{\alpha+\beta\gamma^2}} = \frac{2(c_w + c_r)(\alpha + \beta\gamma^2)}{3\alpha\beta\gamma^2}.$$

This shows that $\Pi_1^{(2)} - \Pi_1^{(3)}$ has one real root, therefore, either $\Pi_1^{(2)} - \Pi_1^{(3)} \geq 0$ or $\Pi_1^{(2)} - \Pi_1^{(3)} \leq 0$ for all $\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)} > 0$. When $\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)} = 0$, $\Pi_1^{(2)} - \Pi_1^{(3)} = \frac{4}{9}(c_w + c_r)^2\frac{\alpha+\beta\gamma^2}{\alpha\beta\gamma^2} > 0$, therefore, $\Pi_1^{(2)} - \Pi_1^{(3)} \geq 0$ for all $\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)} > 0$. Therefore, if $q_{B1}^{(1)} > \gamma q_{A1}^{(1)}$ and $q_{B1}^{(2)} \geq \gamma q_{A1}^{(2)}$, the profit-maximizing quantities for firm 1 are $q_{A1}^{(2)}$ and $q_{B1}^{(2)}$.

We have shown that if $q_{B1}^{(1)} \leq \gamma q_{A1}^{(1)}$, then $q_{A1}^{(1)}$ and $q_{B1}^{(1)}$ (and the corresponding competitor quantities $q_{A2}^{(1)}$ and $q_{B2}^{(1)}$) are optimal (partial conversion). Notice that the condition $q_{B1}^{(1)} \leq \gamma q_{A1}^{(1)}$ can be written as $c_w \leq -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w = -\frac{3\alpha\beta\gamma^2}{2\alpha+\beta\gamma^2}\Delta$. Since $c_w \geq 0$, this condition is only meaningful if $\Delta \leq 0$. Therefore, partial conversion is optimal if and only if $c_w \leq -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$ and $\Delta \leq 0$.

If $q_{B1}^{(1)} > \gamma q_{A1}^{(1)}$ and $q_{B1}^{(2)} \leq \gamma q_{A1}^{(2)}$, then $q_{A1}^{(3)}$ and $q_{B1}^{(3)}$ (and the corresponding competitor quantities $q_{A2}^{(3)}$ and $q_{B2}^{(3)}$) are optimal (exactly-full conversion). The conditions $q_{B1}^{(1)} > \gamma q_{A1}^{(1)}$ and $q_{B1}^{(2)} \leq \gamma q_{A1}^{(2)}$ can be written as $c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w = -\frac{3\alpha\beta\gamma^2}{2\alpha+\beta\gamma^2}\Delta$ and $\frac{1}{3\alpha}c_w + \frac{2(\alpha+\beta\gamma^2)}{3\alpha\beta\gamma^2}c_r \geq \Delta$ (or equivalently, $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} \geq \Delta$), respectively. If $\Delta \leq 0$, then $\frac{2(\alpha+\beta\gamma^2)}{3\alpha\beta\gamma^2}c_r + \frac{1}{3\alpha}c_w \geq \Delta$ holds for any combination of $c_w \geq 0$ and $c_r \geq 0$, making $c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$ the only active condition. If $\Delta > 0$, then $c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$ holds for any $c_w \geq 0$, making $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} \geq \Delta$ the only active condition.

If $q_{B1}^{(1)} > \gamma q_{A1}^{(1)}$ and $q_{B1}^{(2)} > \gamma q_{A1}^{(2)}$, then $q_{A1}^{(2)}$ and $q_{B1}^{(2)}$ (and the corresponding competitor quantities $q_{A2}^{(2)}$ and $q_{B2}^{(2)}$) are optimal (full+ conversion). The conditions $q_{B1}^{(1)} > \gamma q_{A1}^{(1)}$ and $q_{B1}^{(2)} > \gamma q_{A1}^{(2)}$ can be written as $c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w = -\frac{3\alpha\beta\gamma^2}{2\alpha+\beta\gamma^2}\Delta$ and $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < \Delta$, respectively. If $\Delta \leq 0$, then $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < \Delta$ cannot be satisfied for any non-negative values of c_w and c_r. If $\Delta > 0$, then $c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$ holds for any $c_w > 0$, making $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < \Delta$ the only active condition.

To determine whether BPS increases firm profit, notice that the firm's pre-BPS profit is $\Pi_1^{(0)} = (a - \alpha(q_{A1}^{(1)} + q_{A2}^{(1)}) - c_{A1}^o)q_{A1}^{(1)}$. Clearly, if partial conversion is optimal, then profit is $\Pi_1^{(1)} = \Pi_1^{(0)} + (p_B - c_{B1}^o)q_{B1}^{(1)}$, which is greater than $\Pi_1^{(0)}$ if and only if $p_B - c_{B1}^o = \frac{b-2c_{B1}^o+c_{B2}}{3} > 0$. Since $\frac{\partial p_B}{\partial q_{B1}} < 0$, firm 1 can always produce $q_{B1} < q_{B1}^{(1)}$ and still make positive profit. Therefore, if $b - 2c_{B1}^o + c_{B2} > 0$, the firm can always increase profit by implementing BPS and producing $q_{A1} = q_{A1}^{(1)}$ and any quantity $q_{B1} < \min\{q_{B1}^{(1)}, \gamma q_{A1}^{(1)}\}$. If $b - 2c_{B1}^o + c_{B2} \leq 0$, that means the firm cannot make profit in market B with even the lowest cost structure, $c_{B1} = c_{B1}^o$. Therefore, it will not be able to make profit with cost $c_{B1} = c'_{B1} > c_{B1}^o$. Therefore, the firm increases profit by implementing BPS if and only if $b - 2c_{B1}^o + c_{B2} > 0$. This completes the proof. ■

Proof of Proposition 2

We know from (A1) that firm 1's profit can be expressed as $\Pi_1 = \alpha q_{A1}^2 + \beta q_{B1}^2$. When the full+ operating regime is optimal, firm 1's profit is $\Pi_1^{(2)} = \alpha(q_{A1}^{(2)})^2 + \beta(q_{B1}^{(2)})^2$. Differentiating with respect to γ gives:

$$\begin{aligned}\frac{\partial \Pi_1^{(2)}}{\partial \gamma} &= 2\beta q_{B1}^{(2)}\frac{\partial q_{B1}^{(2)}}{\partial \gamma} \\ &= 2\beta q_{B1}^{(2)}\left(\frac{2}{3\beta\gamma^2}(c_{Y1} + c_r)\right) > 0.\end{aligned}$$

When the partial operating regime is optimal, firm 1's profit is $\Pi_1^{(1)} = \alpha(q_{A1}^{(1)})^2 + \beta(q_{B1}^{(1)})^2$. Differentiating with respect to γ gives:

$$\begin{aligned}\frac{\partial \Pi_1^{(1)}}{\partial \gamma} &= 2\beta q_{B1}^{(1)}\frac{\partial q_{B1}^{(1)}}{\partial \gamma} \\ &= 2\beta q_{B1}^{(1)}\left(\frac{2}{3\beta\gamma^2}(c_{Y1} - c_w)\right) > 0 \text{ if and only if } c_{Y1} > c_w.\end{aligned}$$

When the exactly-full operating regime is optimal, firm 1's profit is $\Pi_1^{(3)} = \alpha(q_{A1}^{(3)})^2 +$

$\beta(q_{B1}^{(3)})^2 = (\alpha+\beta\gamma^2)(q_{A1}^{(3)})^2$. We first derive $\frac{\partial q_{A1}^{(3)}}{\partial \gamma}$ (note that $q_{A1}^{(3)} = \frac{a-2c_{A1}^o+c_{A2}+\gamma(b+c_{B2})-2(c_{Y1}-c_w)}{3(\alpha+\beta\gamma^2)}$):

$$\begin{aligned}\frac{\partial q_{A1}^{(3)}}{\partial \gamma} &= \frac{3(b+c_{B2})(\alpha+\beta\gamma^2) - 6\beta\gamma(a-2c_{A1}^o+c_{A2}+\gamma(b+c_{B2})-2(c_{Y1}-c_w))}{9(\alpha+\beta\gamma^2)^2} \\ &= \frac{b+c_{B2}}{3(\alpha+\beta\gamma^2)} - \frac{2\beta\gamma q_{A1}^{(3)}}{\alpha+\beta\gamma^2}\end{aligned}$$

Now we differentiate $\Pi_1^{(3)}$ with respect to γ:

$$\begin{aligned}\frac{\partial \Pi_1^{(3)}}{\partial \gamma} &= 2\beta\gamma(q_{A1}^{(3)})^2 + 2(\alpha+\beta\gamma^2)q_{A1}^{(3)}\frac{\partial q_{A1}^{(3)}}{\partial \gamma} \\ &= 2q_{A1}^{(3)}(\beta\gamma q_{A1}^{(3)} + (\alpha+\beta\gamma^2)\frac{\partial q_{A1}^{(3)}}{\partial \gamma}) \\ &= 2q_{A1}^{(3)}(\beta\gamma q_{A1}^{(3)} + (\alpha+\beta\gamma^2)(\frac{b+c_{B2}}{3(\alpha+\beta\gamma^2)} - \frac{2\beta\gamma q_{A1}^{(3)}}{\alpha+\beta\gamma^2})) \\ &= \frac{b+c_{B2}}{3} - \beta\gamma q_{A1}^{(3)} > 0 \text{ if and only if } \gamma q_{A1}^{(3)} = q_{B1}^{(3)} < \frac{b+c_{B2}}{3\beta}.\end{aligned}$$

We first show that $q_{B1}^{(3)} < q_{B1}^{(1)}$:

$$\begin{aligned}q_{B1}^{(3)} - q_{B1}^{(1)} &= \gamma\rho q_{A1}^{(1)} + (1-\rho)q_{B1}^{(1)} - q_{B1}^{(1)} \\ &= \rho(\gamma q_{A1}^{(1)} - q_{B1}^{(1)}) < 0 \text{ by assumption when exactly-full conversion is optimal.}\end{aligned}$$

Next we show that $q_{B1}^{(1)} < \frac{b+c_{B2}}{3\beta}$ if and only if $c_{Y1} > c_w$:

$$q_{B1}^{(1)} = \frac{b-2c_{B1}^o+c_{B2}}{3\beta} < \frac{b+c_{B2}}{3\beta} \text{ if and only if } c_{B1}^o = \frac{1}{\gamma}(c_{Y1}-c_w) > 0.$$

Since $q_{B1}^{(3)} < q_{B1}^{(1)} < \frac{b+c_{B2}}{3\beta}$ when $c_{Y1} > c_w$, then $c_{Y1} > c_w$ is a sufficient condition for $\frac{\partial \Pi_1^{(3)}}{\partial \gamma} > 0$. This completes the proof. ■

Proof of Lemma 5

If $\Delta \le 0$, the optimal quantities q_{A1}^*, q_{B1}^*, q_{A2}^*, and q_{B2}^* are piecewise linear functions of c_w:

$$q_{A1}^*(c_w) = \begin{cases} q_{A1}^{(1)} = \hat{q}_{A1} - \frac{c_w}{3\alpha} \\ q_{A1}^{(3)} = \rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma} + \frac{c_w}{3(\alpha+\beta\gamma^2)} \end{cases}, \quad q_{B1}^*(c_w) = \begin{cases} q_{B1}^{(1)} = \hat{q}_{B1} + \frac{2c_w}{3\beta\gamma} & , c_w \le -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w \\ q_{B1}^{(3)} = \gamma(\rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma}) + \frac{\gamma c_w}{3(\alpha+\beta\gamma^2)} & , c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w \end{cases}$$

$$q_{A2}^*(c_w) = \begin{cases} q_{A2}^{(1)} = \hat{q}_{A2} - \frac{c_w}{3\alpha} \\ q_{A2}^{(3)} = -\frac{1}{2}(\rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma}) + \frac{a-c_{X2}}{2\alpha} - \frac{(4\alpha+3\beta\gamma^2)c_w}{6\alpha(\alpha+\beta\gamma^2)} \end{cases}, \quad q_{B2}^*(c_w) = \begin{cases} q_{B2}^{(1)} = \hat{q}_{B2} - \frac{c_w}{3\beta\gamma} & , c_w \le -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w \\ q_{B2}^{(3)} = -\frac{\gamma}{2}(\rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma}) + \frac{b-c_{B2}}{2\beta} - \frac{\gamma c_w}{6(\alpha+\beta\gamma^2)} & , c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w \end{cases}$$

Clearly, q^*_{A1}, q^*_{B1}, q^*_{A2}, and q^*_{B2} are independent of c_r. The results $\frac{\partial q^{(1)}_{A1}}{\partial c_w} < 0$, $\frac{\partial q^{(3)}_{A1}}{\partial c_w} > 0$, $\frac{\partial q^{(1)}_{B1}}{\partial c_w} > \frac{\partial q^{(3)}_{B1}}{\partial c_w} > 0$, $\frac{\partial q^{(3)}_{A2}}{\partial c_w} < \frac{\partial q^{(1)}_{A2}}{\partial c_w} < 0$, and $\frac{\partial q^{(1)}_{B2}}{\partial c_w} < \frac{\partial q^{(3)}_{B2}}{\partial c_w} < 0$ can be derived by inspection.

Since $c_w = -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$ is equivalent to the condition $q^{(1)}_{B1} = \gamma q^{(1)}_{A1}$ and we know from Lemma 4 that $q^{(3)}_{A1} = \rho q^{(1)}_{A1} + (1-\rho)\frac{q^{(1)}_{B1}}{\gamma}$, that implies that $q^{(3)}_{A1} = \rho q^{(1)}_{A1} + (1-\rho) q^{(1)}_{A1} = q^{(1)}_{A1}$ when $c_w = -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$. The results $q^{(1)}_{B1} = q^{(3)}_{B1}$, $q^{(1)}_{A2} = q^{(3)}_{A2}$, and $q^{(1)}_{B2} = q^{(3)}_{B2}$ when $c_w = -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$ follow directly. This completes the proof. ■

Proof of Lemma 6

If $\Delta > 0$, the optimal quantities q^*_{A1}, q^*_{B1}, q^*_{A2}, and q^*_{B2} are piecewise linear functions of c_w and c_r:

$$q^*_{A1}(c_w) = \begin{cases} q^{(2)}_{A1} = \hat{q}_{A1} + \frac{2c_r + c_w}{3\alpha} \\ q^{(3)}_{A1} = \rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma} \\ \qquad + \frac{c_w}{3(\alpha+\beta\gamma^2)} \end{cases}, \quad q^*_{B1}(c_w) = \begin{cases} q^{(2)}_{B1} = \hat{q}_{B1} - \frac{2c_r}{3\beta\gamma} & , \frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < 1 \\ q^{(3)}_{B1} = \gamma(\rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma}) & \\ \qquad + \frac{\gamma c_w}{3(\alpha+\beta\gamma^2)} & , \frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} \geq 1 \end{cases}$$

$$q^*_{A2}(c_w) = \begin{cases} q^{(2)}_{A2} = \hat{q}_{A2} - \frac{2c_w + c_r}{3\alpha} \\ q^{(3)}_{A2} = -\frac{1}{2}(\rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma}) \\ \qquad + \frac{a - c_{X2}}{2\alpha} - \frac{(4\alpha+3\beta\gamma^2)c_w}{6\alpha(\alpha+\beta\gamma^2)} \end{cases}, \quad q^*_{B2}(c_w) = \begin{cases} q^{(2)}_{B2} = \hat{q}_{B2} + \frac{c_r}{3\beta\gamma} & , \frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < 1 \\ q^{(3)}_{B2} = -\frac{\gamma}{2}(\rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma}) & \\ \qquad + \frac{b - c_{B2}}{2\beta} - \frac{\gamma c_w}{6(\alpha+\beta\gamma^2)} & , \frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} \geq 1 \end{cases}$$

The results $\frac{\partial q^{(2)}_{A1}}{\partial c_w} > \frac{\partial q^{(3)}_{A1}}{\partial c_w} > 0$, $\frac{\partial q^{(2)}_{A1}}{\partial c_r} > \frac{\partial q^{(3)}_{A1}}{\partial c_r} = 0$, $\frac{\partial q^{(3)}_{B1}}{\partial c_w} > \frac{\partial q^{(2)}_{B1}}{\partial c_w} = 0$, $\frac{\partial q^{(2)}_{B1}}{\partial c_r} < \frac{\partial q^{(3)}_{B1}}{\partial c_r} = 0$, $\frac{\partial q^{(2)}_{A2}}{\partial c_w} < \frac{\partial q^{(3)}_{A2}}{\partial c_w} < 0$, $\frac{\partial q^{(2)}_{A2}}{\partial c_r} < \frac{\partial q^{(3)}_{A2}}{\partial c_r} = 0$, $\frac{\partial q^{(3)}_{B2}}{\partial c_w} < \frac{\partial q^{(2)}_{B2}}{\partial c_w} = 0$, $\frac{\partial q^{(2)}_{B2}}{\partial c_r} > \frac{\partial q^{(3)}_{B2}}{\partial c_r} = 0$ can be derived from inspection.

The condition $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} = 1$ can be written as $c_r = \frac{3\alpha\beta\gamma(\hat{q}_{B1} - \gamma\hat{q}_{A1}) - \beta\gamma^2 c_w}{2(\alpha+\beta\gamma^2)}$. Substituting into $q^{(2)}_{A1}$ gives:

$$\begin{aligned} q^{(2)}_{A1} &= \hat{q}_{A1} + \frac{2c_r + c_w}{3\alpha} \\ &= \hat{q}_{A1} + \frac{2}{3\alpha}\left(\frac{3\alpha\beta\gamma(\hat{q}_{B1} - \gamma\hat{q}_{A1}) - \beta\gamma^2 c_w}{2(\alpha+\beta\gamma^2)}\right) + \frac{c_w}{3\alpha} \\ &= \frac{(\alpha+\beta\gamma^2)\hat{q}_{A1} + \beta\gamma\hat{q}_{B1} - \beta\gamma^2\hat{q}_{A1}}{\alpha+\beta\gamma^2} + \frac{c_w}{3(\alpha+\beta\gamma^2)} \\ &= \frac{\alpha}{\alpha+\beta\gamma^2}\hat{q}_{A1} + \frac{\beta\gamma^2}{\alpha+\beta\gamma^2}\frac{\hat{q}_{B1}}{\gamma} + \frac{c_w}{3(\alpha+\beta\gamma^2)} \\ &= q^{(3)}_{A1} \end{aligned}$$

The results $q^{(2)}_{B1} = q^{(3)}_{B1}$, $q^{(2)}_{A2} = q^{(3)}_{A2}$, and $q^{(2)}_{B2} = q^{(3)}_{B2}$ when $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} = 1$ follow directly. This completes the proof. ■

Proof of Propositions 3 and 4

Market A: Consider first the case when $\Delta \leq 0$. If $c_w \leq -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$, the size of market A is $q_{AT}^{(1)} = q_{A1}^{(1)} + q_{A2}^{(1)} = \hat{q}_{A1} + \hat{q}_{A2} - \frac{2c_w}{3\alpha}$. If $c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$, the size of market A is $q_{AT}^{(3)} = q_{A1}^{(3)} + q_{A2}^{(3)} = \frac{1}{2}(\rho\hat{q}_{A1} + (1-\rho)\frac{\hat{q}_{B1}}{\gamma}) + \frac{a-c_{X2}}{2\alpha} - \frac{2\alpha+3\beta\gamma^2}{6\alpha(\alpha+\beta\gamma^2)}$. Clearly, $\frac{\partial q_{AT}^{(1)}}{\partial c_w} < 0$ and $\frac{\partial q_{AT}^{(3)}}{\partial c_w} < 0$, and both $q_{AT}^{(1)}$ and $q_{AT}^{(3)}$ are constant in c_r. We also know from Lemma 5 that $q_{AT}^{(1)} = q_{AT}^{(3)}$ when $c_w = -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$. Therefore, the size of market A decreases in c_w and is constant in c_r when $\Delta \leq 0$.

Consider now the case when $\Delta > 0$. If $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < 1$, the size of market A is $q_{AT}^{(2)} = q_{A1}^{(2)} + q_{A2}^{(2)} = \hat{q}_{A1} + \hat{q}_{A2} + \frac{c_r - c_w}{3\alpha}$, which clearly decreases in c_w and increases in c_r. If $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} \geq 1$, the size of market A is $q_{AT}^{(3)}$ which we have shown above decreases in c_w and is constant in c_r. From Lemma 6, we know that $q_{AT}^{(2)} = q_{AT}^{(3)}$ when $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} = 1$. Therefore, the size of market A decreases in c_w and weakly increases in c_r when $\Delta > 0$.

Market B: Consider first the case when $\Delta \leq 0$. If $c_w \leq -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$, the size of market B is $q_{BT}^{(1)} = q_{B1}^{(1)} + q_{B2}^{(1)} = \hat{q}_{B1} + \hat{q}_{B2} + \frac{c_w}{3\beta\gamma}$, which increases in c_w and is constant in c_r. If $c_w > -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$, the size of market B is $q_{BT}^{(3)} = \frac{\gamma}{2}(\rho\hat{q}_{A1} + (1-\rho))\frac{\hat{q}_{B1}}{\gamma} + \frac{b-c_{B2}}{2\beta} + \frac{\gamma c_w}{6(\alpha+\beta\gamma^2)}$, which increases in c_w and is constant in c_r. We know from Lemma 5 that $q_{BT}^{(1)} = q_{BT}^{(3)}$ when $c_w = -\frac{\beta\gamma^2}{2\alpha+\beta\gamma^2}\bar{c}_w$. Therefore, the size of market B increases in c_w and is constant in c_r when $\Delta \leq 0$.

Consider now the case when $\Delta > 0$. If $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} < 1$, the size of market B is $q_{BT}^{(2)} = \hat{q}_{B1} + \hat{q}_{B2} - \frac{c_r}{3\beta\gamma}$, which is constant in c_w and decreases in c_r. If $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} \geq 1$, the size of market B is $q_{BT}^{(3)}$ which we have shown above increases in c_w and is constant in c_r. From Lemma 6, we know that $q_{BT}^{(2)} = q_{BT}^{(3)}$ when $\frac{c_w}{\bar{c}_w} + \frac{c_r}{\bar{c}_r} = 1$. Therefore, the size of market B weakly increases in c_w and weakly decreases in c_r when $\Delta > 0$. This completes the proof. ■

Proof of Proposition 5

Partial conversion: When firm 1 implements BPS under partial conversion, the change in emissions is:

$$\begin{aligned} E^{(1)} - E^{(0)} &= -e_w\frac{q_{B1}^{(1)}}{\gamma} + e_Y q_{B1}^{(1)} + e_B(q_{B2}^{(1)} - q_{B2}^{(0)}) \\ &= -e_w\frac{q_{B1}^{(1)}}{\gamma} + e_Y q_{B1}^{(1)} - e_B\frac{q_{B1}^{(1)}}{2}, \end{aligned}$$

which shows that the e_w, e_Y, and e_B terms are negative, positive, and negative, respectively. The e_X and e_r emissions are unchanged.

Full+ conversion: When firm 1 implements BPS under full+ conversion, the change in emissions is:

$$\begin{aligned} E^{(2)} - E^{(0)} &= e_X(q_{A1}^{(2)} + q_{A2}^{(2)} - q_{A1}^{(1)} - q_{A2}^{(1)}) + e_w(q_{A2}^{(2)} - q_{A1}^{(1)} - q_{A2}^{(1)}) \\ &\quad + e_Y\gamma q_{A1}^{(2)} + e_r(q_{B1}^{(2)} - \gamma q_{A1}^{(2)}) + e_B(q_{B2}^{(2)} - q_{B2}^{(0)}) \end{aligned}$$

We show below that the e_X term is positive:

$$\begin{aligned} q_{A1}^{(2)} + q_{A2}^{(2)} - q_{A1}^{(1)} - q_{A2}^{(1)} &= q_{A1}^{(1)} + \frac{2(c_w + c_r)}{3\alpha} + q_{A2}^{(1)} - \frac{c_w + c_r}{3\alpha} - q_{A1}^{(1)} - q_{A2}^{(1)} \\ &= \frac{c_w + c_r}{3\alpha} > 0 \end{aligned}$$

We show below that the e_w term is negative:

$$q_{A2}^{(2)} - q_{A1}^{(1)} - q_{A2}^{(1)} = q_{A2}^{(1)} - \frac{c_w + c_r}{3\alpha} - q_{A1}^{(1)} - q_{A2}^{(1)} = -\frac{c_w + c_r}{3\alpha} - q_{A1}^{(1)} < 0$$

We show below that the e_B term is negative:

$$q_{B2}^{(2)} - q_{B2}^{(0)} = \frac{b - 2c_{B2} - c_{B1}^o}{3\beta} + \frac{c_w + c_r}{3\beta\gamma} - \frac{b - c_{B2}}{2\beta} = -\frac{q_{B1}^{(1)}}{2} + \frac{c_w + c_r}{3\beta\gamma} = -\frac{q_{B1}^{(2)}}{2} < 0$$

The e_r term is positive by assumption when full+ conversion is optimal. The e_Y term is positive by inspection.

Exactly-full conversion: When firm 1 implements BPS under exactly-full conversion, the change in emissions is:

$$\begin{aligned} E^{(3)} - E^{(0)} &= e_X(q_{A1}^{(3)} + q_{A2}^{(3)} - q_{A1}^{(1)} - q_{A2}^{(1)}) + e_w(q_{A2}^{(3)} - q_{A1}^{(1)} - q_{A2}^{(1)}) \\ &\quad + e_Y q_{B1}^{(3)} + e_B(q_{B2}^{(3)} - q_{B2}^{(0)}) \end{aligned}$$

We show below that the e_X term is positive:

$$\begin{aligned} & q_{A1}^{(3)} + q_{A2}^{(3)} - q_{A1}^{(1)} - q_{A2}^{(1)} \\ = \ & \rho q_{A1}^{(1)} + (1-\rho)\frac{q_{B1}^{(1)}}{\gamma} + \rho q_{A2}^{(1)} + (1-\rho)(\frac{a - c_{A2}}{2\alpha} - \frac{q_{B1}^{(1)}}{2\gamma}) - q_{A1}^{(1)} - q_{A2}^{(1)} \\ = \ & \frac{1-\rho}{2}(\frac{q_{B1}^{(1)}}{\gamma} - q_{A1}^{(1)}) > 0 \text{ by assumption when exactly full conversion is optimal.} \end{aligned}$$

We show below that $q_{A2}^{(3)} - q_{A2}^{(1)} < 0$ which is sufficient to show that the e_w term is negative:

$$\begin{aligned} q_{A2}^{(3)} - q_{A2}^{(1)} &= \rho q_{A2}^{(1)} + (1-\rho)(\frac{a - c_{A2}}{2\alpha} - \frac{q_{B1}^{(1)}}{2\gamma}) - q_{A2}^{(1)} \\ &= (1-\rho)(\frac{a - c_{A2}}{2\alpha} - q_{A2}^{(1)} - \frac{q_{B1}^{(1)}}{2\gamma}) \\ &= \frac{(1-\rho)}{2}(q_{A1}^{(1)} - \frac{q_{B1}^{(1)}}{\gamma}) < 0 \text{ by assumption when exactly full conversion is optimal.} \end{aligned}$$

We show below that the e_B term is negative:

$$\begin{aligned}
q_{B2}^{(3)} - q_{B2}^{(0)} &= (1-\rho)q_{B2}^{(1)} + \rho(q_{B2}^{(0)} - \frac{\gamma q_{A1}^{(1)}}{2}) - q_{B2}^{(0)} \\
&= (1-\rho)q_{B2}^{(1)} + (\rho-1)q_{B2}^{(0)} - \frac{\rho\gamma q_{A1}^{(1)}}{2} \\
&= (1-\rho)(\frac{b-2c_{B2}-c_{B1}^{o}}{3\beta} - \frac{b-c_{B2}}{2\beta}) - \frac{\rho\gamma q_{A1}^{(1)}}{2} \\
&= -\frac{1}{2}((1-\rho)q_{B1}^{(1)} + \rho\gamma q_{A1}^{(1)}) < 0.
\end{aligned}$$

The e_Y term is positive by inspection. This completes the proof. ■

Proof of Corollary 1

We know from Proposition 5 that e_B emissions from competitor $B2$ decreases. Proposition 5 also shows that e_X emissions from competitor $A2$ remain unchanged under partial conversion. We show below that e_X emissions from competitor $A2$ decreases under full+ and exactly-full conversion:

$$\begin{aligned}
\text{full+: } q_{A2}^{(2)} - q_{A2}^{(1)} &= q_{A2}^{(1)} - \frac{c_w + c_r}{3\alpha} - q_{A2}^{(1)} < 0 \\
\text{exactly-full: } q_{A2}^{(3)} - q_{A2}^{(1)} &= \rho q_{A2}^{(1)} + (1-\rho)(\frac{a-c_{A2}}{2\alpha} - \frac{q_{B1}^{(1)}}{2\gamma}) - q_{A2}^{(1)} \\
&= (1-\rho)(\frac{a-c_{A2}}{2\alpha} - q_{A2}^{(1)} - \frac{q_{B1}^{(1)}}{2\gamma}) \\
&= (1-\rho)(\frac{a-c_{A2}}{2\alpha} - \frac{a-2c_{A2}+c_{A1}^{o}}{3\alpha} - \frac{q_{B1}^{(1)}}{2\gamma}) \\
&= \frac{1-\rho}{2}(q_{A1}^{(1)} - \frac{q_{B1}^{(1)}}{\gamma}) \\
&< 0 \text{ by assumption when exactly-full conversion is optimal.}
\end{aligned}$$

We have shown that emissions from competitors $A2$ and $B2$ (weakly) decrease after firm 1 implements BPS. Therefore, if firm 1 decreases its own emissions after implementing BPS, then total emissions decrease. This completes the proof. ■